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13

AN

ACCOUNT

OF THE

CALCULATIONS

MADE FROM THE

SURVEY AND MEASURES

TAKEN AT

SCHEHALLIEN,

IN ORDER TO ASCERTAIN THE

MEAN DENSITY OF THE EARTH.

BY CHARLES HUTTON, ESQ. F.R.S.

Read at the ROYAL SOCIETY, May 21, 1778.

LONDON:
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AN ACCOUNT, &c.

THE ſurvey from which theſe calculations have been made was taken at and about the hill Schehallien in Perthſhire, in the years 1774, 1775, and 1776, by the direction, and partly under the inſpection, of the Rev. NEVIL MASKELYNE, D. D. F. R. S. and Aſtronomer Royal, by whom the manner of making the ſurvey has already been fully explained in the Philoſophical Tranſactions for 1775.

I have therefore only to give an account of the meaſures of the lines and angles, and of the calculations which I have raiſed from them with all poſſible care and faithfulneſs, for the purpoſe of determining the meaſure of the ratio of the mean denſity of the earth to that of water or any other known matter.

Theſe calculations were naturally and unavoidably long and tedious; and the more ſo as the buſineſs was in a manner quite new, which laid me under the neceſſity of inventing and deſcribing ſuch modes of computation as ſhould be proper to be applied in ſo important and delicate a buſineſs. Having, at length, with cloſe and unwearied application for a conſiderable time compleated all the calculations; I have, in the following ſheets, drawn up an account of thoſe operations, with the reſults ariſing from them; and have accompanied them with ſuch drawings as are neceſſary to illuſtrate the deſcriptions. I have alſo inſerted a ſynopſis of the meaſures which were taken of the lines and angles, from which any perſon may at any time ſatisfy himſelf of the truth of the computations that have been made and are herein deſcribed. Theſe meaſures I have here immediately ſubjoined, before I proceed to deſcribe the computations made from them.

A ſynopſis of the horizontal and vertical angles that were obſerved at the principal points in making the ſurvey about Schehallien.

In the firſt column are contained the names of the horizontal angles, the meaſure of which, in degrees and minutes, are in the ſecond column, and the vertical angles are in the third column, in which it is to be obſerved, that the letter denoting the object is placed before the degrees and minutes, and E or D after them, to ſhew that they are in elevation or depreſſion reſpectively. The mark * placed to the meaſure of any angle denotes that it is the mean of the two obſervations made with the inſtrument turned different ways; namely, after the firſt obſervation, reverſing it to make the ſecond. Alſo the mean height of the theodolite is put down to each ſtation. In the vertical angles, the bottom of the object is underſtood, unleſs where the top is mentioned below, and ſometimes the height of the pole is added in feet and inches.

At A	Theodolite = 4 ft. 10 in.	
	° ′	° ′
DAB	31 15	
DAN	77 30	N 12 22E
DAO	102 36½	O 0 58E
DAR	134 31½	
NAC	83 13½	N 12 20E
NAO	25 4	
OAS	27 24	S 5 25D
OAR	31 55	R 6 31D
At B		
DBN	81 8	
DBO	101 41	
DBA	139 59	
At C	Theodolite = 4 ft. 7½ in.	
ACD	126 6	
ACF	93 34	
ACG	92 15	
ACH	85 31½	
ACN	49 10	N 10 4E
ACB	8 11	
At D	Theodolite = 4 ft. 8½ in.	
ADC	48 7	
ADB	8 45	
ADN	49 14	N 10 40E
ADW	83 56½	N 10 39E
ADH	88 56	A 1 15E
ADG	95 5	G 3 17D
ADF	96 3	F 3 34D

At F	Theodolite = 4 ft. 8½ in.	
	° ′	° ′
GFW	17 27	W 0 58 D
GFK	76 7¼	K 9 50 E
GFN	103 57½	N 17 42½E
GFD	172 54	D 3 37 E
HFG	10 9	G 1 36 D
HFP	60 41	P 5 9 E
HFK	65 58	K 9 41 E
HFN	93 49	N 17 35½E
At G	Theodolite = 4 ft. 6½ in.	
DGF	6 8	
NGD	59 41	
NGF	65 49	
NGH	101 9	
HGF	166 57	
HGD	160 49	
HGX	71 11½	
HGW	4 32	
PGF	95 56	F 1 15 E
PGN	30 6½	N 16 54 E top of Cairn.
PGK	0 9¼	K 10 15 E
PGL	62 57	L 0 32 D
PGW	63 20	W 0 54 D
PGH	71 2	H 3 0 D P 5 46 E top of ball.

At

At H	Theodolite = 4 ft. 5½ in.	
	° ′	° ′
GHF	2 54	G 2 59 E
GHD	13 2	
NHG	49 44	N 14 42 E
NHK	29 17	
NHW	107 43	N 14 39 E top
NHW	107 41	
KHD	65 59	K 12 6 E top
KHG	79 1	
KHW	78 25	W 3 2 E
WHP	96 14	P 7 39 E top
WH*p*	149 2	*p* 0 22 E
WHG	157 25½	G 2 55 E
At W	Theodolite = 4 ft. 5½ in.	
LWK	107 27½	L 0 25½ E
LWP	130 40	P 5 43 E top
LWP	130 41	
LWN	133 48½	N 12 36½ E
LWF	175 2	K 11 30 E
LWQ	4 41	
LWG	178 21	
LWH	193 13	H 3 14 D
At L	Theodolite = 4 ft. 9 in.	
GLP	37 1	P 4 24½ E
GLY	139 32½ *	Y 2 35½ D
WLG	1 18	W 0 * 40½ D
	1 17¾ *	G 0 * 31 E
WLP	38 12	P 4 * 22½ E
	38 12½ *	
WLN	38 58	N 10 * 56½ E
	38 58½ *	
WLK	60 33	K 10 * 24½ E
	60 34 *	
WLT	115 38 *	T 0 * 30 D
WLV	126 58½ *	V 0 * 35½ D
WLY	140 50½ *	Y 2 * 35 D
WLF	3 58½ *	F 0 * 38½ E
WLE	62 12 *	E 10 * 16½ E
WL*t*	38 4½ *	*t* 4 * 34½ E
At Y	Theodolite = 4 ft. 8 in.	
TYK	69 23¼ *	K 10 * 29 E
TYN	76 56½ *	N 9 * 37¼ E
TYL	124 47 *	L 2 * 35 E
TYG	103 24¾ *	G 1 * 36½ E
TYZ	19 21¼ *	Z 3 * 3½ E
TYV	13 29½ *	V 3 * 26½ E
VYT	13 28 *	T 2 * 4½ E
VYZ	32 52 *	Z 3 * 3½ E
VYE	81 49½ *	E 10 * 33 E
VYK	82 53 *	K 10 * 29 E
VYN	90 24 *	N 9 * 36½ E
VYG	116 52 *	G 1 * 35½ E
VYH	126 57 *	H 1 * 0 E
VYW	130 33 *	W 2 * 2 E
VYL	138 14 *	L 2 * 33½ E

At

At v	Theodolite = 4 ft. 11 in.		
	° ′		° ′
TVZ	17 25¼ *	Z	2 * 28½ E
	17 25 *		2 * 26
TVU	34 24½ *	U	2 * 47½ E
TVE	70 43 *	E	9 * 10 E
TVK	71 23½ *	K	9 * 5½ E
	71 24 *		9 * 5½
TVW	112 32½ *	W	0 * 25½ E
TVL	119 57 *	L	0 * 34 E
TVY	147 51½ *	Y	3 * 31½ D
	147 50 *		3 * 28½ D
		T	0 6 E
At T	Theodolite = 4 ft. 10 in.		
YTV	18 41½ *	V	0 * 8¾ D
YTL	30 1 *	L	0 * 27¼ E
YTU	116 18¼ *	U	3 * 31 E
YTZ	133 31½ *	Z	3 * 49½ E
		Y	2 10 D
VTY	18 41 *	Y	2 * 6 D
VTL	48 43 *	L	0 * 30 E
VTE	94 2 *	E	9 * 42 E
VTU	135 1½ *	U	3 * 31½ E
VTZ	152 14½ *	Z	3 * 50 E
		V	0 7 D
At z	Theodolite = 4 ft. 6½ in.		
UZR	10 53½ *	R	0 * 50 D
UZA	16 5 *	A	1 * 58¾ E
UZX	16 18 *	X	1 * 52 D
UZO	33 18¾ *	O	3 * 58 E
UZY	117 21 *	Y	3 * 8¼ D
UZV	134 4½ *	V	2 * 31½ D
UZT	144 27½ *	T	3 * 56¼ D
		U	2 * 45¼ E
At U	Theodolite = 4 ft. 10 in.		
ZUT	18 20¾ *	T	3 * 32¼ D
ZUV	28 54½ *	V	2 * 51 D
ZUO	124 42 *	O	4 * 1½ E
ZUN	125 9½ *	N	10 * 13½ E
ZUX	128 2¾ *	X	10 * 55½ D
ZUA	157 55 *	A	1 * 36¾ E
ZUR	160 37½ *	R	3 * 38 D
		Z	2 * 51 D
At x	Theodolite = 4 ft. 7 in.		
OXA	40 59½ *	A	4 * 15 E
OXS	53 20 *	S	0 * 44¼ E
OXZ	139 22½ *	U	10 * 49¼ E
OXU	174 59 *	Z	1 * 48¼ E
		O	11 * 13½ E
At s	Theodolite = 4 ft. 9 in.		
RSA	18 27	A	5 24 E
RSA	18 28	A	5 20½ E
RSN	72 51	N	18 40 E
RSO	90 58	O	13 19½ E
RSX	171 1	X	0 55½ D
		R	0 24½ E

At R the E. end of the S. baſe.	Theodolite = 4 ft. 6 in.	
	° ′	° ′
KRN	41 6½	
B″RA	20 36¼ *	A 6 * 27½ E
B″RN	84 32¾ *	N 19 * 17½ E
B″RO	111 46¼ *	O 11 * 57¼ E
B″Rα′	100 14½ *	α′ 20 * 2 E
B″Rβ′	121 27¼ *	β′ 15 * 11¼ E
B″RS	177 39	
B″RZ	186 53½ *	Z 0 * 46 E
its ſup.	173 6½ *	O 12 29 E

At B″ the W. end of the S. baſe.	Theodolite = 4 ft. 10 in.	
RB″S	0 55½	R 0 37 D
RB″β′	36 8¼ *	β′ 10 * 24 E
RB″O	37 27 *	O 7 * 35¾ E
RB″α′	50 49½ *	α′ 15 * 48 E
RB″N	65 46¾ *	N 17 * 33 E
RB″A	139 2¼ *	A 11 * 22¼ E

At O the S. obſervatory.	Theodolite = 4 ft. 7 in.	
	° ′	° ′
XOS	35 39 *	S 13 * 25½ D
XOM	42 12¾ *	M 13 * 12½ D
XOR	58 49¼ *	R 12 * 4 D
XOB″	89 35 *	B″ 7 * 39½ D
XOA	115 43½ *	A 0 * 59 D
XOU	1 39 *	U 4 * 4½ D
XOZ	23 38½ *	Z 4 * 0 D K 11 * 18 D
MOR	16 36½ *	R 12 * 3½ D
MOB″	47 23 *	B″ 7 * 39½ D
MOA	73 32½ *	A 1 * 0½ D
MOB	81 22 *	B 1 * 1½ D top
MON	134 39	M 13 16
NOK	95 1	
MOα′	156 34 *	α′ 33 * 47½ E top
MOβ′	125 45 *	β′ 21 * 29½ E
MOZ	65 51½ *	Z 4 * 0 D
MOU	43 51½ *	U 4 * 3½ D
MOK	42 13½ *	K 11 * 19 D
MOS	6 34½ *	S 13 * 22½ D

M is in the meridian paſſing through O, and is a little South of the interſection of that meridian and the line RS.

At P the N. obſervatory.	Theodolite = 4 ft. 10 in.	
NPK	77 18¾	
GPK	179 35¼ *	G 5 39¾ D
GPH	47 45¼ *	H 7 * 32½ D
GPW	69 0¼ *	W 5 * 37 D
GPL	80 9¼ *	L 4 * 27¾ D
GPF′	20 54 *	F′ 5 * 6 D
GP*m*	38 18¼ *	*m* 12 * 20 D
γ′Pδ	45 39	} by reduction
γ′PK	2 55¾	

γ′ from P bears 30° 41′⅖ E. of South.
δ from P bears 14 57 W. of South.
m is Mr. MASON's mark.

At

At N the Weſt-ern cairn.	Theodolite = 4 ft. 11 in. ° ′	° ′
KNO	40 5¼	by reduction
KNR	74 48⅖	
KNP	50 47⅘	
KNɣ	9 37½	
KNγ′	2 11⅖	
KNA	133 53 *	A 12 * 22½ D
KNB	146 15½ *	B 11 * 17 D top
KNC	178 30½ *	C 10 * 4 D
KND	172 51 *	D 10 * 39 D top
KN*n*	144 37½ *	*n* 4 * 30½ D
KNG	98 23 *	G 16 * 55 D top Pole 4 ft. 4 in.
KNH	69 13⅓	H 14 38 D top Pole 6 ft. 5 in.
KN*m*	64 27 *	*m* 18 * 55 D
KNW	56 22 *	W 12 * 35 D top Pole 7 ft.
KNL	49 7 *	L 10 * 55 D top Pole 6 ft. 8 in.
KNY	18 50 *	Y 9 * 37½ D top
KNγ′	2 15 *	γ′ 7 * 30 D
KNE	1 15½ *	E 6 * 38½ D
KNɣ	9 37½ *	ɣ 6 * 25½ D
KNU	41 4½ *	U 10 * 12½ D top
KNS	60 44 *	S 18 * 41 D
KNR	74 48½ *	R 19 * 20 D
KNB″	104 30 *	B″ 17 * 35½ D
		K 6 * 43½ D top

At K the Eaſt-ern cairn.	Theodolite = 4 ft. 8 in. ° ′	° ′
NKO	44 51¾	by reduction
NKR	64 5⅓	
NKP	51 54⅓	N 6 * 41 E
NK*n*	0 45 *	*n* 6 * 36 E top
NKF	43 32½ *	F 9 * 46 D top Pole 4 ft. 2 in.
NKG	51 40 *	G 10 * 16 D top Pole 4 ft. 4 in.
		G 10 * 14½ D
NKα	55 12	α 10 3 D
NKH	81 28	H 12 7 D top Pole 6 ft. 5 in.
NKH	81 27½ *	H 12 * 5½ D
NKW	97 18 *	W 11 * 28 D top Pole 7 ft.
NKL	109 19¾ *	L 10 * 25 D top Pole 6 ft. 8 in.
NKL	109 18½ *	L 10 * 23½ D
NKY	153 40 *	Y 10 * 30 D top
NKV	174 20 *	V 9 * 5½ D top
NKE	156 18	E 3 34 D top
NKα′	11 36 *	α′ 6 * 31¾ E
NKM′	7 8 *	M′ 4 * 51¼ E
NKɣ	6 2⅖ *	ɣ 6 * 53 E
*n*Kα	54 26⅕ *	α 10 3¼ D
*n*Kα	54 25½ *	α 10 * 3½ D
*n*Kβ	39 16½ *	β 8 * 58½ D
α′Kβ′	37 39	α′ 6 32 E

At

At α the E. end of the N. baſe.	Theodolite = 4 ft. 4 in.	
	° ′	° ′
γαK	106 0 *	K 10* 1½E
γαG	101 3 *	G 9*37½E
γαN	89 41 *	N 13*45½E
γα*n*	89 25 *	*n* 13*49½E
γαF′	47 38 *	F′ 8* 6½E
		β 0 19 D
		γ 0 0 top Pole 3 ft. 2 in.
At β between α and γ the ends of the N. baſe.	Theodolite = 4 ft. 8 in.	
αβ*k*	174 51	*k* 0 41½E
αβ*t*′	147 37½	*t*′ 1*25½E
αβF′	71 49 *	F′ 10*22⅔E
αβD	108 1¼ *	D 9*17½E
αβ*n*	70 58½ *	*n* 13* 5⅔E
γβD	71 57¾ *	D 9*17$\frac{7}{10}$E
γβF′	108 9½ *	F′ 10*22¾E
		α 0* 6 D
At γ the Weſtern end of the North baſe.	Theodolite = 4 ft. 8½ in.	
αγF′	50 6½ *	F′ 8*23½E
αγ*n*	63 53½ *	*n* 12*27¼E
αγD	97 5⅓ *	D 9*41 E
		α 0* 2 D top Pole 3 ft. 2 in.

At *n* the new W. cairn.	Theodolite = 4 ft. 9 in.	
	° ′	° ′
K*n*x	108 59½ *	K 6*40½D
K*n*β	128 35¾ *	β 13* 7½D
K*n*F′	128 55½ *	F′ 14*10½D
K*n*γ	135 40½ *	γ 12*29 D
K*n*D	173 55	D 10*45 D
K*n*N	38 28½ *	N 1 42 D
K*n*B″	102 54¼ *	B″17 31 D
D*n*A	53 49 *	A 12*21½D
D*n*G	75 24 *	G 17* 3½D
D*n*P″	78 44¼	P″20* 2½D P″ is a pole in a line with P and K
D*n*H	104 35	H 14*42½D
D*n*L	124 34	L 10 56 D
D*n*K	173 55	α 13*51⅜D
N*n*G	132 59	α 13*54 D N*n* was = 93½ feet by the tape meaſure.
At E the new E. cairn.	Theodolite = 4 ft. 9 in.	
NEA	10 5½ *	A 6*22½E
NEM′	4 58½ *	M′ 4*45½E
NED	4 51½ *	D 6*41½E
NEK	22 14½ *	K 2*11½E
NEH	78 40½ *	H 11*47½D top Pole 6 ft. 5 in.
NEW	94 17 *	W 11*16½D top Pole 7 ft
NEL	106 21½ *	L 10*17 D top Pole 6 ft. 8 in

	° ′	° ′
NEY	151 16 *	Y 10*37 D
NEV	172 20 *	V 9*15 D
NET	172 22½ *	T 9*41½ D top
NEZ	144 46 *	Z 9*18½ D
NEU	114 20½ *	U 10* 4½ D
a′E*α*′	68 35½ *	*α*′ 6*21 E
a′Eγ′	73 50½ *	γ′ 6*37½ E
a′EN	78 39½ *	N 6*33 E
a′E*b*′	97 17 *	*b*′ 9* 5½ D
		a′ 8*27 D

At M′ the meridian mark on the top of the hill South of P.	Theodolite = 3 ft. 10½ in.	
KM′E	3 28½ *	E 4*56½ D
KM′γ′	6 37½ *	γ′ 6* 3½ D top
KM′L	53 27½ *	L 10*19 D top Pole 6 ft. 8 in.
KM′W	63 12½ *	W 11*41 D top Pole 7 ft.
KM′H	78 4½ *	H 12*56½ D top Pole 6 ft. 5 in.
KM′*m*	86 33½ *	*m* 22*12 D
KM′*p*	87 54 *	*p* 22*10 D
KM′G	108 49 *	G 12*33 D top Pole 4 ft. 4 in.
KM′F	118 3½ *	F 12*25 D top Pole 4 ft. 2 in.
KM′δ′	176 1½ *	δ′ 12* 8 E
		K 5 5 D

At *t* the center of the tranſit inſtrument near the N. obſerv.	Theodolite = 4 ft. 8½ in.	
	° ′	° ′
M″*t*P′	32 24 *	P′ 9*18½ D
M″*t*G	32 57½ *	G 5*58½ D
M″*tm*	5 14 *	*m* 12*32½ D
M″*t*H	14 28 *	H 7*44½ D
M″*t*W	35 23½ *	W 5*47½ D
M″*t*L	46 22 *	L 4*35½ D
mtp	68 13 *	*p* 12*50½ E
*mt*M′	68 15 *	M′ 22* 5 E
*mt*G	144 17 *	G 5*57½ D
*mt*H	96 52 *	H 7*43½ D
*mt*W	75 56 *	W 5*47½ D
*mt*L	64 58 *	L 4*37½ D
*mt*P	54 4½ *	P 17*42½ D top
		m 4* 3 E

M″ bears 1′ 8″ W. of North.
M′ bears South.
P′ in a line with K and P.
p, a pole immediately above or South of the tranſit inſtrument.

At *p*	Theodolite = 4 ft. 5 in.	
G*p*M′	147 5 *	M′ 22* 8½ E
G*p*γ′	133 6½ *	γ′ 24*18 E
G*p*F	26 43	F 5 26 D
G*p*P′	0 34½ *	P′ 9*23½ D
G*p*M″	32 47¾ *	M″ 13*25½ D
G*pt*	32 48	*t* 19 17 D
G*pm*	38 0½ *	*m* 12*34½ D
G*p*H	47 11 *	H 7*49½ D

G*p*W

	° ′	° ′
GpW	68 3½ *	W 5 * 51½ D
GpP	76 55½ *	P 13 * 49½ D
GpL	79 0½ *	L 4 * 39½ D
Gpm′	121 24 *	m′ 4 * 50½ D
GpE	179 37 *	E 17 * 39½ E
m′ is a pole a little above or South of P.		
At m′		
Gm′M′	145 4½ *	M′ 22 * 8½ E
Gm′δ′	131 11 *	δ′ 24 * 12½ E
Gm′p	57 53½ *	p 2 * 47 D
Gm′t	34 43	t 10 15 D
Gm′F	13 1 *	F 5 * 22½ D
Gm′P′	0 29½ *	P′ 19 * 19 D
Gm′M″	32 29½ *	M″ 13 * 23½ D
Gm′P	37 6 *	P 18 * 16½ D
Gm′m	37 49½ *	m 12 * 36½ D
Gm′H	47 10 *	H 7 * 49 D
Gm′W	68 11 *	W 5 * 49½ D
Gm′L	79 17½ *	L 4 * 40½ D
Gm′E	177 53 *	E 17 * 54 E
At m		
PmK	15 57	K 15 0 E
Pma	37 11	P 12 27 E
At α′ the South-weſt pole.	Theodolite = 4 ft. 11 in.	
Kα′N	149 3 *	K 6 * 43½ D
Kα′B′	131 3 *	B′ 15 * 51½ D
Kα′R	102 7¼ *	R 20 * 9 D
Kα′O	73 39 *	O 33 * 52 D
Kα′β′	28 4¼ *	β′ 15 * 56½ D top
Kα′γ′	4 53⅝ *	γ′ 7 * 35⅛ top
Kα′δ	65 24 *	δ 3 41½ top Pole 17 ft. 4 in.
Eα′δ′	13 48 *	δ′ 12 * 54½ D
Eα′a′	8 36½ *	a′ 8 * 3½ D
Eα′K	2 49½ *	E 6 30 D
At β′ the South-eaſt pole.	Theodolite = 4 ft. 9 in.	
α′β′O	56 47 *	O 21 * 45½ D
α′β′B′	62 21½ *	B′ 10 * 28½ D
α′β′R	84 46 *	R 15 * 17 D
α′β M	96 41 *	M 17 * 3½ D
α′β′K	114 8½ *	α′ 15 * 42½ E
At γ′ the North-eaſt pole.	Theodolite = 4 ft. 4 in.	
Nγ′P	56 55¾ *	P 18 * 3⅝ D
Nγ′t	55 22½	t 17 49½ D
Nγ′n	0 50⅞ *	n 7 15 E
Nγ′δ′	8 17¾ *	δ′ 7 * 59¼ E
Nγ′α′	14 17 *	α′ 7 * 31 E
Nγ′K	151 54½ *	K 1 * 34½ E
		N 7 * 23½ E

At δ′ the North-weſt pole	Theodolite = 4 ft. 4 in.	
	° ′	° ′
Kδ′M′	2 53 *	M′ 12 * 36½ D
Kδ′γ′	4 29 *	γ′ 8 * 8½ D
Kδ′*m*′	73 20½ *	*m*′ 24 * 9½ D top
Kδ′*p*	74 35½ *	*p* 24 * 16½ D top
Kδ′α	108 48½ *	α 12 * 8½ D top Pole 4 ft. 4 in.
Kδ′N	164 19¼ *	N 6 * 8½ D top
Kδ′α′	108 56 *	K 7 * 3 D
Kδ′E	2 30¼ *	E 6 * 49 D
Eδ′*a*	9 5½ *	*a* 8 * 50½ D
γ′δ′α′	113 20½ *	
γ′δ′*t*	70 14½ *	*t* 24 20½ D
At F′	Theodolite = 4 ft. 7½ in.	
DF′*n*	56 24$\frac{7}{10}$ *	
DF′P	77 37½ *	
DF′α	174 40¼ *	α 8 * 12⅘ D
DE′β	124 47 *	β 10 * 30⅓ D
DF′γ	103 4 *	γ 8 * 28⅗ D
DF′*t*′	77 42	D 7 * 11 E
At *t*′	Theodolite = 4 ft. 10 in.	
β*t*′F′	57 7	β 1 * 37½ D
β*t*′*k*	115 32	*k* 0 41 D
F′*t*′*k*	172 38	F′ 7 * 50½ E

At *k*	Theodolite = 4 ft. 10½ in.	
	° ′	° ′
t′*k*F′	4 42	F 5 11 E
t′*k*α	34 47	α 0 27 D
t′*k*β	37 13	β 0 52 D
		t′ 0 26 E
At *a*	Theodolite = 4 ft. 8½ in.	
*ba*K	113 57½	K 14 45 E Top of the cairn.
*ba*N	155 24	
*ba*W	80 40	
*ba*L	59 48	
	Theodolite = 4 ft. 6 in.	
At *b*	43 35	K 13 32 E
*ab*K	18 49	N 11 26
*ab*N		*a* 5 39
At *d*	Theodolite = 4 ft. 9 in.	
*cd*N	14 11	N 11 33 E
*cd*G	34 53	
*cd*H	63 27	H 6 36 D
*cd*L	111 12	L 6 42 D
At *c*	Theodolite = 4 ft. 8 in.	
*dc*L	49 34 *	L 7 * 21½ D
*dc*H	97 9 *	H 9 * 17½ D
*dc*G	136 9½ *	G 4 * 58 D
*dc*K	118 55½ *	K 14 * 35½ E
*dc*F	144 20	F 4 24 D

At *a′*	Theodolite = 4 ft. 10 in.	
	° ′	° ′
E*a′α′*	102 48 *	*α′* 7*56 E
E*a′γ*	97 5½*	*γ′* 8*10 E
E*a′b′*	65 58½*	*b′* 6* 4½D
E*a′c′*	88 6 *	*c′* 11*20½D
E*a′d′*	108 21½*	*d′* 2*34½E
		E 7 19E
At *b′*	Theodolite = 4 ft. 9½ in.	
c′b′a′	36 41½*	*a′* 5*44½E
*c′b′*E	53 26½*	E 8*41½E
		c′ 4 41 D

At *c′*	Theodolite = 4 ft. 8½ in.	
	° ′	° ′
a′c′b′	121 12½*	*b′* 6* 1 E
		a′ 9* 6 E
At *d′*	Theodolite = 4 ft. 5 in.	
a′d′α′	169 14½*	*α′* 12*35½E
		a′ 2 48 D

Several other angles and bearing of objects were taken, which, being of no uſe in computing the attraction of the hill, are here omitted.

The foregoing tables, containing all the angles collected together which were obſerved at the ſame point, include all the horizontal angles that were at different times taken for aſcertaining the relative places of the principal points and objects on a horizontal plane. The numerous other angles uſed, in finding the ſections of the ground, are given hereafter, with their computed reſults annexed to them.

We now proceed to ſpeak of the two principal baſes which were accurately meaſured, as foundations on which every thing elſe muſt depend; and firſt, *Of the meaſure of the baſe* RB″ *in* Glenmore, *the valley on the South of* Schehallien, *taken the* 16*th*, &c. *of* Sept. 1774.

Here A and B are the names of the two meaſuring rods, which were laid down alternately in the order as expreſſed in the following table of meaſures. The lengths of theſe rods, by the braſs ſtandard, when the thermometer was at 62¾, were thus, *viz.*

$$\left\{\begin{array}{l} \mathrm{A} = 20 \text{ feet } 1\cdot255 \text{ inch.} = 20\cdot10458 \\ \mathrm{B} = 20 \text{ feet } 1\cdot323 \text{ inch.} = 20\cdot11025 \end{array}\right\} \text{feet.}$$

The numbers following each rod, with the ſign + interpoſed, are inches and decimal parts; and they denote the diſtance beyond the end of each rod to the beginning of the next following rod; and, therefore, the ſum of all theſe numbers muſt be added to the ſum of the lengths of the rods themſelves for the total of the meaſures. Alſo, as the firſt rod began at 2 feet 8 inches from the point R, this number is to be added to the total laſt mentioned, to give the meaſure of the whole baſe from R to B″.

A + 8·29

A +8·29
B +2·53
A +6·11
B +6·66
A +2·79
B +1·20
A +2·07
B +4·80
A +0·00
B +1·78
A +3·29
B +2·85
A +6·39
B +4·86
A +6·08
B +8·58
A +9·07
B +1·58
A +2·28
B +7·47
A +2·40
B +5·42
A +7·42
B +8·14
A +8·77

B +3·45
A +3·80
B +6·64
A +7 76
B +3·28
A +4·87
B +6·18
A +8·70
A +7·87
B +4·75
A +6·56
B +5·24
A +7·90
B +6·32
A +6·92
B +7·28
A +6·34
B +8·93
A +5·39
B +5·20
A +3·54
B +1·26
A +3·20
B +5·34
A +3·74

B +7·22
A +1·91
B +4·46
A +1·95
B +2·26
A +4·54
B +4·48
A +3·14
B +3·38
A +5 00
B +4·85
A +6·12
B +3·44
A +6·18
B +4·19
A +4·51
B +3·04
A +4·37
B +2·96
A +2·47
B +3·90
A +5·78
B +3·97
A +4·87
B +4·83

A +3·34
B +3·65
A +6·96
B +3·07
A +3·55
B +2·93
A +5·53
B +5·33
A +4·38
B +3·67
A +5·12
B +1·06
A +5·96
B +2·47
A +3·84
B +5·57
A +2·63
B +7·41
A +3·11
B +1·74
A +2·07
B +4·33
A +5·93
B +6·36

A +4·47
B +3·75
A +4·74
B +3·06
A +2·58
B +4·15
A +3·16
B +4 64
A +3·26
B +4·18
A +4·04
B +2·92
A +3·10
B +5·11
A +4·61
B +3·34
A +2·57
B +5·80
A +3·37
B +2·58
A +2·24
B +3·48
A +2·95
B +2 88

A +3·69
B +4·07
A +2·75
B +4·65
A +4·07
B +3·23
A +4·16
B +5·73
A +4·12
B +4·91
A +3·18
B +3·91
A +5·28
B +2·90
A +4·39*
B +4·37
A +3·29
B +2·12
A +2·95
B +3·30
A +2·82
B +3 97
A +1·37
B +0·00

The ſum of all theſe is 74A+73B+669·28 inches, or 74A+73B+55·773 feet, including the 2 feet 8 inches at the beginning of the meaſurement.

Now 74A is = 1487·73892
73B is = 1468·04825
The odd parts 55·773
Sum 3011·56017 = the baſe unreduced.

But a reduction of this muſt be here made according to the ſtate of the thermometer, and for the wearing of the braſs 5 feet ſtandard (ſee Phil. Tranſ. vol. LVIII. for the year 1768, p. 313, &c.). Now the difference between 62° and $62\frac{3}{4}$ being $\frac{3}{4}$, therefore 3011·56 $\times \frac{232}{1800000 \times 12} \times \frac{3}{4} = 3011{\cdot}56 \times \frac{29}{2700000} \times \frac{3}{4} = 0{\cdot}024$ feet, is the ſmall correction on account of the thermometer, and which being added makes the number become 3011·584 for the length of the baſe as reduced to the ſtate of 62° of FAHRENHEIT's thermometer. But the braſs rod had been $\frac{1}{1000}$th of an inch ſhortened by wearing, and it was originally $\frac{1}{1000}$th of an inch ſhorter than the Royal Society's braſs ſtandard yard, ſo that it is now $\frac{1}{500}$dth inch ſhorter than that ſtandard in the length of 3 feet, or $\frac{1}{18000}$th part of the whole; therefore ſubtracting the $\frac{1}{18000}$th part, or ·167 from the above quantity, there remains 3011·417 feet for the corrected meaſure of this baſe, or the true length of the line RB''.

The above meaſures, as far as to that marked ∗ incluſively, together with 10 feet $10\frac{1}{2}$ inches more, reach to a place to which they had before meaſured with the tape line, and by it found to be 2844·8 feet; while the meaſure of the ſame by the rods is found to be 2839·3 feet. The difference is 5½ feet, a ſmall part of which might be owing to the unſtable ſtate of the wooden ſtands uſed in the firſt quarter of the baſe; but the greater part of this difference is more likely to be owing to the uncertain way of meaſuring with a tape, which, to ſay nothing of the ground not being quite level, is liable to be ſtretched more or leſs in length with different degrees of tenſion, and to be variouſly warped in length by moiſture.

Of the meaſurement of the baſe $\alpha\beta\gamma$ *in* Rannoch, *to the North-weſt of the hill of* Schehallien.

1. One part of this baſe was meaſured twice over in different ways. The part $\alpha\beta$ was carefully meaſured on the 8th of October 1774 with a chain, and found to be 63 chains and 40½ links, or 63·405 chains in length.

Now on the 24th of the ſame month the chain was meaſured by means of the five-feet braſs ſtandard, when the thermometer was at 38°½, and the length found to be 65·94542 feet. Hence then $65{\cdot}94542 \times 63{\cdot}405 = 4181{\cdot}269$ is the length of all the chains, to which, adding 1·764 the breadth of the 63 iron pins, the ſum is 4183·033 for the length of $\alpha\beta$ uncorrected.

But $62-38\frac{1}{2}=23\frac{1}{2}$, therefore $-23\frac{1}{2}\times\frac{29}{2700000}\times4183$ $=-1·056$ is the reduction on account of the ſtate of the thermometer, which being applied with its proper ſign, there reſults 4181·977; and from this laſt number deducting again $\frac{1}{18000}$th part or ·232, on account of the wearing of the braſs ſtandard, there then remains 4181·745 feet for the length of the part $\alpha\beta$ of the baſe in Rannoch, as meaſured by the chain.

But as the chain was meaſured not at the ſame time with the baſe, but between two and three weeks later, when the air was probably cooler, the reduction above made for the ſtate of the thermometer is perhaps ſomething too great, and we may ſafely conclude $\alpha\beta$ to be equal 4182 feet as meaſured by the chain.

2. The whole baſe $\alpha\beta\gamma$ was next, on the 10th, 11th, and 12th of October, very carefully meaſured by the twenty-feet meaſuring rods. The rods at that time meaſured thus,

$$\left\{\begin{array}{l}\text{A} = 20 \text{ ft.} + 1·306 \text{ inch.} = 20·108\frac{5}{6}\\ \text{B} = 20 \text{ ft.} + 1·354 \text{ inch.} = 20·112\frac{5}{6}\end{array}\right\} \text{feet};$$

the thermometer being then at 40°. The number of rods and the additional parts were as follows.

A +4·49	A +3·99	B +1·73	B +3·22	B +2·43	B +1·87
B +3·29	B +3·13	A +2·41	A +3·16	A +1·97	A +2·38
A +6·57	A +3·55	B +2·99	B +2·07	B +1·74	B +2·67
B +3·62	B +4·19	A +2·62	A +2·22	A +2·84	A +2·07
A +3·84	A +4·06	B +2·18	B +4·83	B +3·65	B +1·84
B +3·52	B +3·94	A +2·72	A +2·39	A +1·62	A +4·03
A +4·50	A +3·64	B +3·02	B +2·68	B +1·27	B +1·72
B +3·62	B +3·23	A +2·46	A +2·09	A +2·38	A +2·14
A +4·88	A +3·76	B +3·68	B +1·97	B +2·36	B +1·80
B +2·74	B +2·56	A +2·62	A +1·48	A +2·57	A +1·27
A +3·24	A +3·38	B +2·72	B +3·20	B +2·07	B +1·88
B +4·30	B +3·29	A +3·33	A +2·62	A +2·48	A +2·36
A +3·50	A +3·65	B +2·93	B +2·37	B +2·31	B +2·04
B +3·26	B +3·51	A +3·14	A +2·47	A +2·28	A +2·37
A +2·96	A +3·88	B +2·93	B +3·48	B +3·96	B +1·77
B +3·32	B +2·29	A +2·40	A +3·16	A +4·87	A +1·66
A +5·93	A +3·13	B +2·06	B +3·50	B +2·61	B +2·26
B +5·00	B +3·71	A +3·13	A +2·33	A +2·22	A +1·97
A +3·43	A +3·13	B +2·57	B +2·37	B +1·63	B +1·77
B +3·87	B +3·13	A +2·85	A +2·68	A +1·87	A +2·63
A +6·37	A +5·43	B +2·87	B +2·68	B +2·41	B +3·18
B +3·48	B +3·08	A +4·12	A +2·70	A +2·62	A +2·09
A +4·86	A +3·57	B +2·20	B +2·68	B +2·09	B +1·74
B +4·87	B +5·68	A +2·66	A +2·05	A +2·27	A +2·74
A +3·08	A +3·89	B +2·17	B +3·09	B +3·02	B +4·49
B +3·67	B +2·78	A +2·13	A +2·50	A +2·41	A +2·60
A +3·28	A +3·27	B +2·17	B +2·52	B +2·53	B +2·45
B +3·43	B +1·84	A +3·12	A +2·62	A +1·84	A +1·41
A +4·82	A +0·00	B +2·70	B +2·43	B +2·68	B +2·67
B +4·46	A +3·14	A +3·17	A +3·01	A +2·37	A +1·98

B +2·96
A +2·27
B +2·60
A +3·08
B +2·23
A +4·70
B +2·28
A +1·79
B +1·83
A +2·74
B +2·48
A +2·23
B +1·66
A +3·08
B +2·31
A +2·20
B +3·06
A +2·36
B +2·24
A +1·66
B +1·53
A +1·80
B +3·53
A +2·45
B +0·00*
A +2·36
B +2·75
A +1·77
B +1·55
A +1·97
B +2·04
A +2·56
B +2·15
A +2·26
B +2·37
A +1·94
B +1·97
A +1·94
B +2·77
A +2·09
B +2·14
A +2·45
B +2·98
A +2·40
B +2·50
A +2·82
B +2·37
A +2·76
B +2·91
A +2·58
B +2·34
A +2·86
B +2·67
A +2·19
B +2·37
A +2·93
B +2·37
A +2·86
B +2·42
A +2·16
B +2·21
A +2·75
B +2·03
A +2·73
B +2·14
A +1·57
B +1·77
A +2·21
B +2·11
A +2·99
B +2·06
A +2·36
B +1·77
A +1·66
B +1·86
A +2·33
B +2·65
A +2·14
B +2·35
A +2·43
B +2·59
A +2·48
B +2·48
A +2·91
B +2·47
A +1·80
B +1·99
A +2·81
B +2·44
A +2·07
B +2·44
A +2·11
B +2·49
A +2·11
B +3·03
A +2·67
B +2·43
A +1·93
B +2·75
A +1·99
B +2·17
A +1·83
B +1·93
A +1·65
B +1·04
A +1·96
B +2·77
A +2·25
B +1·79
A +0·00

Of the foregoing meaſures, the ſum of all from the beginning to that marked * incluſively, together with 13 feet 2 inches more, brings us to the point β before meaſured to by the chain. Now to this place, by adding together the meaſures, there are found to be 103 A and 102 B, and the ſum of the parts is 586·71 inches.

Then $103\text{A} = 103 \times 20{\cdot}108\tfrac{5}{6} = 2071{\cdot}210$

$102\text{B} = 102 \times 20{\cdot}112\tfrac{5}{6} = 2051{\cdot}509$

586·71 inches $= 48{\cdot}893$

13 ft. 2 inch. $= 13{\cdot}167$

Hence $\alpha\beta$ (unreduced) is $4184{\cdot}779$

But ſince $62 - 40 = 22$, therefore the reduction for the ſtate of the air is $-22 \times \frac{29}{2700000} \times 4185 = -{\cdot}989$, which being applied to the above ſum, there remains 4183·79 as correſponding to the ſtate of 62° of the thermometer. From this laſt number deduct its $\frac{1}{18000}$th part, *viz.* ·232, and there reſults 4183·558 for the correct length of the part $\alpha\beta$ as determined by this very accurate method; which is but about a foot and a half more than what it was found to be by the leſs accurate meaſure by the chain, which is a nearer approach to an equality than could well be expected.

To determine now the whole length of the baſe $\alpha\gamma$; by taking the whole ſums there are found to be 146 A with 144 B and 779·78 inches of the odd parts.

Then $146\text{A} = 146 \times 20{\cdot}108\tfrac{5}{6} = 2935{\cdot}890$

$144\text{B} = 144 \times 20{\cdot}112\tfrac{5}{6} = 2896{\cdot}248$

779·78 inches $= 64{\cdot}982$

The ſum or $\alpha\gamma$ (unreduced) is $5897{\cdot}120$

The correction for the thermometer is $-22 \times \frac{29}{2700000} \times 5897 = -1{\cdot}394$, which being applied to the number above, there reſults 5895.726; and this again being diminiſhed by its $\frac{1}{18000}$th part, or ·327, there remains 5895·399 feet, for the correct meaſure of the baſe $\alpha\gamma$ in the vale of Rannoch.

There is no occaſion here to explain the manner of meaſuring theſe two baſes by the twenty-foot rods, as that has been very circumſtantially done in vol. LXV. of the Phil. Tranſ. for the year 1775, by the rev. Dr. MASKELYNE, the learned and accurate conductor of this very important experiment.

The following ſhorter lines were alſo meaſured as they happened to be wanted in different parts of the ſurvey.

	Feet.	Inch.	
$\alpha'\delta'$ =	269	4	nearly horizontal.
Nn =	93	6	
Ke =	94	10	
KE =	240	10	
ac =	9	9	
an =	7	10	
cn =	1	11	
ma =	70	11	not horizontal.
mt =	68	3	
mp =	63	4	
pt =	27	2	

The other meaſures that were taken for determining the ſections will be delivered afterwards, when the reſults or computed altitudes have been obtained, in order to be placed oppoſite to their correſpondent angles.

Having now obtained, to a great degree of accuracy, the meaſured lengths of two lines which were to ſerve as baſes for all the future calculations, the next conſideration was how to make the propereſt uſe of them. Every other line or diſtance, drawn or conceived to be drawn, muſt be calculated from them by the help of the angles obſerved either at their extremities, or at all the other points and ſtations in the ſurvey and plan. As theſe two baſes are ſituated in the low parts of the country, from whence but a very few of the other principal ſtations are viſible, one method evidently is to compute immediately from theſe baſes ſuch of the great lines in the ſurvey whoſe extremities are viſible from them; and then from theſe calculated lines to compute others next to them, and ſo on quite around and within the whole figure. In this manner ſeveral values of each line will ariſe, both from the double computations by the two meaſured baſes, and from the various ſets of triangles which can be formed from the very numerous horizontal angles which were obſerved at the ſeveral ſtations. But in this mode of computation, after great labour and pains, I had frequently the mortification to find that the ſeveral values of the ſame lines would differ ſo greatly one from another, that I was often very doubtful whether I could rely on any of them, or even on the mean among them all. Theſe differences aroſe from the ſmall errors in the obſerved angles, which in ſome degree are unavoidable; and indeed they were ſo ſmall, that the ſum of the angles of the ſeveral triangles which were uſed in the calculation ſeldom differed by more than a minute or two from 180°. But in a long connected chain of triangles, dependant on one another, the effects of ſuch ſmall errors at length become too great to be tolerated in a computation requiring much accuracy. Another method is, firſt to compute from both baſes the length of the line KN extended along the ridge of the hill from Eaſt to Weſt, and from it, as a ſecondary baſe, compute all the other lines in the plan. This method admits of much more accuracy than the former, ſuppoſing this ſecondary baſe to be truly aſſigned; becauſe that, from the elevated and central ſituation of this line, all or moſt of the other points in the ſurvey are viſible from one or both of its extremities, by which it happens that the other lines are moſtly determinable from it alone, without ſo cloſe a connection with one another as in the other method of computation. By both of theſe methods then, and by all the triangles furniſhed by each of them, I computed all the principal lines in the plan, and either took a mean among the ſeveral values of each, or elſe ſelected out of them ſuch one as from various circumſtances I judged it ſafeſt to rely upon, as neareſt the truth. The trigonometrical computations were always accurately made, and generally repeated by logarithms, and the reſult of every proportion determined to two or three places of decimals. I ſhall here abſtract the mean or corrected values of ſome of the principal lines or horizontal diſtances ſo computed, as well as the ſecondary baſe KN from the Eaſtern to the Weſtern cairn.

The mean among a great number of ways of computation from the South baſe gives the horizontal diſtance from K to N = 4052·2, and the mean of all the reſults from the North baſe $\alpha\beta\gamma$ gives KN = 4058·9, and the mean between theſe two gives 4055·5 for the mean diſtance of K and N. And this value of KN was uſed in computing moſt of the other lines, whoſe mean reſults are as here follows.

$\alpha\gamma$ = 5895·4 the Northern baſe in Rannoch.
RB$''$ = 3011·4 the Southern baſe in Glenmore.
NK = 4055·5 the diſtance of the two cairns.

RA = 5670
AB = 1489
BC = 4506
CD = 775
DF = 7388
FG = 1166
GH = 4068
HW = 2118
WL = 1816
LY = 7085
YV = 3636
VT = 2645
TZ = 4393
ZU = 4132
UX = 1984
XS = 2378
SR = 1410

NR = 5545
NB$''$ = 6053
NA = 5941
NB = 6573
NC = 7797
ND = 7657
NF = 5980
NG = 6370
NH = 8195
NW = 9059
NL = 10405
NY = 13752
NS = 5795
NO = 2875
NP = 3271
Nα = 11876
Na = 5899
Nb = 7614
Nb = 3381
Nδ' = 1585

KR = 5952
KF = 8227
KG = 8036
KH = 7748
KW = 7603
KL = 8335
KY = 10008
KV = 10215
KO = 2615
KP = 3221
Kα = 13710
Kβ = 15404
KM$'$ = 1817
Kδ' = 2528
Ka = 3326
Kb = 4409

PG = 4815
PH = 5196

OR = 3582
OB$''$ = 5466
OA = 6769
OS = 3271
OX = 4079
OU = 6061
OZ = 9073
OM = 3317

MS = 381
Te = 1335
ze = 3719
F$'$D = 6430
F$'$F = 3934
F$'t'$ = 4098
$t'k$ = 2327
M$'b$ = 1172
ab = 1843
cd = 1750

From the three firſt lines, or baſes, and the horizontal angles obſerved at the ſeveral ſtations, a very large and accurate plan of the whole ſurvey was conſtructed, forming a map of four feet long by four feet broad, which was verified in every part by the meaſures of the computed lines, both thoſe above-written and others, and they were generally found to agree very exactly, according to the ſcale by which the plan was conſtructed. The uſe of this large map was to receive and admit of the diſtinct and accurate exhibition of the figures in their true places, expreſſing the number of feet in elevation or depreſſion with reſpect to each obſervatory of every point and ſection of the ground whoſe elevation or depreſſion might be obſerved. But before I proceed to the computation and conſtruction of the points in the ſections, I ſhall here abſtract the numbers which expreſs the relative elevation of the principal original points in the ſurvey, being the extremes of the lines whoſe lengths are above abſtracted. Theſe few numbers are the reſults of the calculation of ſeveral hundreds of triangles conceived in a vertical poſition, their baſes being either the horizontal lines above-written, or other lines drawn as diagonals between many diſtant points in the ſurvey, according to the number of vertical angles which had been obſerved; and of theſe baſes, whether real or imaginary, each generally afforded two vertical triangles, as the angles of elevation and depreſſion were taken alternately at both ends of the lines. It is ſcarcely neceſſary to remark, that all theſe triangles are right-angled, the common baſe being one of the ſides about the right angle, and the other the difference in altitude between the two given points or extremes of the baſe; and this difference in altitude is found from the application of this proportion, as radius is to the tangent of the angle of elevation or depreſſion, ſo is the given baſe to the altitudinal difference between the two given points, excluſive of the height of the theodolite or other inſtrument, which was afterwards allowed for. From the reſolution of all theſe triangles, and taking the means of the many correſponding reſults, were obtained the following numbers, which ſhew how many feet the points denoted by the letters ſtanding againſt them are below the level of the point N or the Weſtern cairn. They are all referred to this point N at the Weſtern extremity of the ridge of the hill, becauſe it is the moſt elevated point in the whole ſurvey.

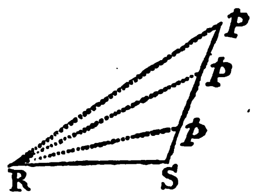
O	1184	γ	2898	H	2143	U	1613	*e*	2145
P	1457	A	1303	W	2024	X	1996	M	1958
K	480	B	1313	L	2006	S	1964	M′	322
R	1948	C	1384	Y	2335	*a*	1012	F′	2246
B″	1920	D	1445	V	2119	*b*	823	t'	2815
α	2898	F	1904	T	2114	*c*	1364	*k*	2835
β	2901	G	1935	Z	1815	*d*	1539	δ'	172

Theſe depreſſions, and thoſe of ſeveral other principal points, were firſt carefully computed by means of various different baſes, as ſo many places from whence the ſections were to commence.

Theſe ſections are very numerous, made in all directions from the primitive points before mentioned, and many of them extended to great diſtances far beyond the bounds of the plan hereunto annexed, ſo as to include the neareſt hills and valleys of the ſurrounding country. They are moſtly made in vertical planes in the manner deſcribed in the article of the Phil. Tranſ. before referred to, excepting ſome few of them which are level ſections in planes parallel to the horizon, and ſome indeed irregular as being neither vertical nor horizontal. To compute the relative altitude of each point in theſe ſections, it is evident, requires the reſolution of two different triangles, *viz.* a horizontal triangle by which its place in the plan is aſcertained, and a vertical triangle of which one ſide is the elevation or depreſſion of the point. Of theſe ſections there are above 70, containing near 1000 points, whoſe places in the plan and relative altitudes have been computed: ſo that the number of triangles, whoſe numerical reſolutions have been performed in the courſe of this buſineſs, amounts to ſeveral thouſands.

Before the abſtract of the computation of the ſections, I ſhall here put down at large the calculation of one of them, to ſhew the manner in which they have been computed in the readieſt and eaſieſt way that occurred to me, preſerving at the ſame time the proper degree of accuracy.

racy. I ſhall for this purpoſe ſelect the third ſection as not containing ſo many poles as ſome of the others. This ſection commences at S, and is carried up the hill in a vertical plane, making an angle of 105° with the line RS. The direction of this plane is here repreſented by the line S*ppp* making with RS the angle RSp=105°. The points *ppp* &c. mark the places of the poles, whoſe angles of elevation or depreſſion were taken at S with a proper inſtrument, and they are written in the ſecond column of the table in this example. At R were obſerved the ſeveral horizontal angles, which lines ſuppoſed to be drawn from thence made with RS, and theſe are placed in the third column. And ſince in every triangle RSp, the angle S is conſtant, and the ſum of R and p is equal to the conſtant quantity 75°: therefore each of the angles R, or the numbers in the third column, being ſubtracted from 75°, there remains the correſponding angle p: and theſe remainders are placed in the fourth column. Then, ſince the method of ſolution is this, as ſ.p : ſ.R :: RS : Sp=$\frac{\text{ſ.R}}{\text{ſ.}p}$ × RS; and again, as radius (1) : tang. elev. :: Sp : alt. of p above S = Sp × tang. elev. = $\frac{\text{ſ.R}}{\text{ſ.}p}$ × RS × tang. elev. Or in logarithms



From — 180°
Take ∠S= 105
Leaves ∠R+∠p= 75

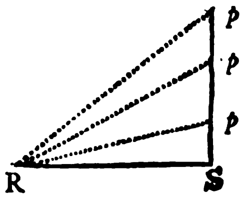
ſ.R − ſ.p + RS + tang. elev. = log. of the altitude of the point. Wherefore having taken, from a table, the ſines of R and p, and placed them in the fifth and ſixth columns, ſubtract the latter from the former, and write the remainders in the next or ſeventh column; to theſe add the conſtant logarithm of RS, and write the ſums in the eighth column; take out then the tangents of the angles in the ſecond column, and having placed them in the ninth column, add together the adjacent numbers of the eighth and ninth columns, placing the ſums in the tenth column, which being the logarithms of the altitudes or depreſſions of the points p, take the correſponding numbers from a table of logarithms, and write them in the eleventh or laſt column, for thoſe altitudes or depreſſions with reſpect to the point S, with the height of the theodolite included, and which is afterwards allowed for, its height being generally about $4\frac{1}{2}$ or $4\frac{3}{4}$ feet. In the ſecond column D denotes depreſſion and E elevation; in the laſt column D denotes depreſſion and A altitude.

N° of Poles.	Angles of Dep. and Elev. at S.	Horiz. Ang. at R.	75° − ∠'s R, or angles p.	Sines of R.	Sines of p.	Sin. R − ſin. p.	Sin. R − ſin. p + RS, or sp.	Tang. of Dep. and Elev.	Sum of 8 and 9, or Log. Dep. and Alt.	Dep. and alt. below or above S.
	° ′	° ′	° ′							
1	3 27D	19 11	55 49	9·51666	9·91763	9·59903	2·74843	8·78022	1·52865	34D
2	2 36E	30 24	44 36	9·70418	9·84643	9·85775	3·00715	8·65715	1·66430	46A
3	4 33	38 28	36 32	9·79383	9·77473	0·01910	3·16850	8·90080	2·06930	117
4	6 12	44 10	30 50	9·84308	9·70973	0·13335	3·28275	9·03597	2·31872	208
5	7 51	47 55	27 5	9·87050	9·61828	0·25222	3·40162	9·13948	2·54110	348
6	10 38	51 22	23 38	9·89574	9·60302	0·29272	3·44212	9·27357	2·71569	520
7	12 20	53 9	21 51	9·90320	9·57075	0·33245	3·48185	9·33974	2·82159	663
8	13 46	54 43	20 17	9·91185	9·53991	0·37194	3·52134	9·38918	2·91052	814
9	15 43	56 21	18 39	9·92035	9·50486	0·41549	3·56489	9·44933	3·01422	1033
10	17 35	57 47	17 13	9·92739	9·47127	0·45612	3·60552	9·50092	3·10644	1278
11	18 0	58 58	16 2	9·93291	9·44122	0·49169	3·64109	9·51178	3·15287	1422
1	2	3	4	5	6	7	8	9	10	11

Thus then every line in the table contains the ſolutions of the two triangles, the one horizontal and the other vertical, uſed in finding the altitude of each point or pole in the ſection. The addition of the conſtant logarithm of the baſe RS to the logarithms in the ſeventh column, is moſt eaſily performed by writing it on the bottom of a little ſlip of paper, and ſo ſliding it down ſucceſſively over each of thoſe numbers, and in that poſition adding them together, and placing the ſums immediately oppoſite in the next column.

And in this manner were computed the relative altitudes of the points in the other vertical ſections; excepting two or three caſes, in which the conſtant angle formed by the ſection and the baſe was a right angle; and one caſe in which the vertical angles were not taken at the beginning of the ſection line, but at the other end of the baſe line where the horizontal angles were alſo obſerved. It may be neceſſary, therefore, to inſert and explain an example of each of theſe caſes, and the more ſo as they point out the propereſt means of meaſuring theſe ſections ſo as to ſave moſt part of the labour in the computation, in which the trouble chiefly conſiſts.

Of the caſe of the right angle, the firſt ſection is an inſtance, where alſo RS is the baſe as before, and the angle RSp being = 90°.

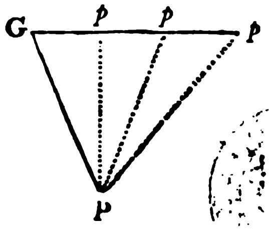


Poles.	Dep. and Elev. at S.	Horiz. Ang. at R.	Tang. of vert. ∠'s at S.	Tang. of ∠'s R.	Sum of Columns 4 and 5.	6th + RS = Log. Alt.	Depth and Alt.
	° ′	° ′					
1	5 16½ D	10 0	8·96533	9·24632	8·21165	1·36105	23 D
2	0 30 E	31 35	7·94086	9·78874	7·72960	0·87899	7½ A
3	4 15	41 56	8·87106	9·95342	8·82448	1·97388	94
4	6 14¼	49 25	9·03861	10·06722	9·10583	2·25523	180
5	8 16	55 51½	9·16224	10·16870	9·33094	2·48033	302
6	10 13	59 57½	9·25582	10·23783	9·49365	2·64305	440
7	11 37	62 56½	9·31297	10·29174	9·60471	2·75411	568
8	12 25	65 3¾	9·34276	10·33257	9·67533	2·82472	668
9	13 21	66 41½	9·37532	10·36568	9·74100	2·89040	777
10	14 10	67 36¾	9·40212	10·38519	9·78731	2·93671	864
11	15 17	68 42½	9·43657	10·40925	9·84582	2·99522	989
12	17 46	70 58	9.50572	10.46221	9·96793	3·11733	1310
13	19 33	72 48	9·55035	10·50927	10·05962	3·20902	1618
14	20 6	74 30	9·56342	10·55701	10·12043	3·26983	1861
1	2	3	4	5	6	7	8

In this form there are three columns leſs than in the former, by which it happens, that about one-third of the labour is ſaved. The method of ſolution is thus; as 1 (radius) : tang. R :: RS : S*p* = RS × t. R; and again, as 1 : tang. S (vertical angle) :: S*p* : S*p* × t. S = RS × t. R. × t. S. Or, in logarithms, log. RS + t. R + t. S = log. of the vertical perpendicular: and by this theorem, it is evident, the columns of this table are conſtructed.

But nearly the ſame ſaving in the great labour of computation would be made if the vertical and horizontal angles had both been taken at the end of the baſe fartheſt from the beginning of the ſection. And this method would alſo be much the eaſieſt in making the ſurvey on the ground, as there would then need only one obſerver with an inſtrument to meaſure both horizontal and vertical angles; and any perſon, without an inſtrument, could direct in a line the perſon who moves and places the poles, or he may even direct himſelf after his firſt pole has been placed, by means of a back object, as is commonly done in land ſurveying.

Of this kind there happens to have been one ſection taken, proceeding from G, and making with GP an angle of 85°, P being the Northern obſervatory, and where both the bearings and depreſſions of the points p in the ſection line were obſerved.



$$\begin{array}{rr} & 180 \\ \angle G = & 85 \\ \hline \angle P + \angle p = & 95 \end{array}$$

$$\left.\begin{array}{lr} & \text{Log.} \\ \text{PG} & 3{\cdot}68262 \\ \text{ſ.G} & 9{\cdot}99834 \\ \hline \text{Sum} & 3{\cdot}68096 \\ \hline \end{array}\right\}$$

which is a conſtant number from which the ſines of p in the fifth column are to be deducted.

Poles.	Vertic. Angles at P.	Horizont. Ang. at P.	95° − ∠P = ∠*p*	Sines of ∠'s at *p*.	PG + *s*.G − *s*.*p* = P*p*	Tang. of Depr.	Sum of Col. 6 and 7. = P*p*.	Dep. below P.
	° ′	° ′	° ′					
1	8 45D	9 15	85 45	9·99880	3·68216	9·18728	2·86944	740
2	8 46	16 25	78 35	9·99132	3·68964	9·18812	2·87776	755
3	9 58	27 16	67 44	9·96634	3·71462	9·24484	2·95946	911
4	8 38	30 52	64 8	9·95415	3·72681	9·18136	2·90817	809
5	7 6	34 30	60 30	9·93970	3·74126	9·09537	2·83663	686
6	5 23	37 55	57 5	9·92400	3·75696	8·97421	2·73117	538
1	2	3	4	5	6	7	8	9

Here it is evident is a ſaving of two of the moſt laborious columns in the table. This happens becauſe that in every triangle PG*p* there are now conſtant thoſe two parts which are uſed in the proportion made uſe of in the calculation, *viz.* PG and the angle G. For then it is, as ſ.*p* : ſ.G :: PG : P*p*, or log. P*p*=log. PG+ſ.G−ſ.*p*; ſo that the ſum of the logarithms of PG and ſine of ∠ G is a conſtant number, from which the numbers in the fifth column are to be ſubtracted, to find thoſe in the ſixth column. The reſt of the work is the ſame as in the firſt example.

As to the irregular ſections, the computation of them differs ſo little in manner from that of the uſual vertical ſections, that an example of it is unneceſſary: and the few horizontal ſections need no computation, but only an allowance for the height of the theodolite.

In the following abſtract of the reſults of the computation of the ſections, the firſt column contains the number of the pole, the ſecond and third the vertical and horizontal angles, and the laſt the difference of altitude in feet, between the foot of each pole and the point from whence the vertical angles were obſerved, after making the allowance for the height of the theodolite above the ground. At the end of this abſtract is a plate of the figures referring to the number of the ſection, ſhewing the direction in which it was carried, with the degrees and minutes in the angle formed by it and the baſe line.

SECTION 1.

Pole.	Vert. ∠'s at s.	Bearings at R.	Diff. of Alt.
	° '	° '	
1	5 16½ D	10 0	18 D
2	0 30 E	31 35	12 A
3	4 15	41 56	99
4	6 14¼	49 25	185
5	8 16	55 51½	307
6	10 13	59 57½	444
7	11 37	62 56½	572
8	12 25	65 3¾	673
9	13 21	66 41½	782
10	14 10	67 36¾	869
11	15 17	68 42½	994
12	17 46	70 58	1315
13	19 33	72 48	1623
14	20 6	74 30	1866

SECTION 2.

Pole.	Vert. ∠'s at s.	Bearings at R.	Diff. of Alt.
	° '	° '	
1	3 40 D	20 46	30 D
2	1 26 E	32 42	28 A
3	4 20	42 8	103
4	6 21	49 30	192
5	9 58½	59 2	429
6	11 50½	62 30	591
7	12 52	65 2	721
8	13 22	65 51	780
9	13 36½	66 9	806
	° '	° '	
10	15 12½	67 36	972
11	16 24	68 45	1118
12	17 45	70 0	1302
13	18 45	71 3	1467
14	19 35	71 57½	1625
15	19 59	72 34	1726

SECTION 3.

Pole.	Vert. ∠'s at s.	Bearings at R.	Diff. of Alt.
	° '	° '	
1	3 27 D	19 11	29 D
2	2 36 E	30 24	51 A
3	4 33	38 28	122
4	6 12	44 10	213
5	7 51	47 55	352
6	10 38	51 22	524
7	12 20	53 9	668
8	13 46	54 43	818
9	15 43	56 21	1038
10	17 35	57 47	1283
11	18 0	58 58	427

SECTION 4.

Pole	Vert. ∠'s at R.	Bearings at s.	Diff. of Alt.
	° '	° '	
1	7 42 D	13 8	39 D
2	1 2 E	31 35	19 A
3	4 4	40 24	80
4	5 36	48 36	137
5	6 55	56 56	215

	° ′	° ′	
6	9 10	62 36	337
7	9 23	68 12	415
8	10 17	70 33	495
9	11 18	74 0	623
10	13 2	76 3	785
11	13 37	76 47	848
12	14 30	77 45	946
13	15 3	78 58	1042
14	16 15	80 11	1200
15	17 24	81 13	1362
16	18 32	82 5	1528
17	19 29	82 59	1699
18	20 7	83 30	1812

SECTION 5.

Pole.	Vert. ∠'s at s.	Bearings at x.	Diff. of Alt.
	° ′	° ′	
1	3 46 D	15 0	38 D
2	0 14 E	24 23	9 A
3	2 16	36 14	60
4	3 29	46 5	112
5	4 21	54 24	163
6	5 48	63 55	258
7	7 2	71 30	361
8	9 3	76 30	513
9	11 25	81 18	720
10	12 55	84 16	874
11	13 54	87 22	1017
12	14 51	90 30	1182
13	15 9	93 0	1295

SECTION 6.

Pole.	Vert. ∠'s at x.	Bearings at s.	Diff. of Alt.
	° ′	° ′	
1	9 16 D	11 43	80 D
2	4 46 D	16 51	60 D
3	1 10 E	23 50	28 A
4	4 51	30 2	137
5	6 6	33 14	196
6	7 32	37 25	287
7	9 30	40 24	409
8	11 15	43 12	546
9	12 22	45 18	656
10	13 35	47 9	782
11	15 19	48 54	955
12	16 29	50 12	1093
13	17 8	51 1	1171
14	17 27	51 48	1248

SECTION 7.

Pole.	Vert. ∠'s at x.	Bearings at s.	Diff. of Alt.
	° ′	° ′	
1	7 30 D	9 30	60 D
2	1 49	15 52	24
3	2 5 E	20 49	52 A
4	4 30	24 48	135
5	5 41	28 23	208
6	7 13	30 27	297
7	8 14	32 29	380
8	9 24	33 41	465
9	10 26	35 0	558

	° ′	° ′	
10	11 41	36 21	678
11	12 36	37 15	773
12	13 31	38 17	885
13	13 46	39 30	973
14	13 56	40 17	1036

SECTION 8.

Pole.	Vert. ∠'s at A.	Bearings at B.	Diff. of Alt.
	° ′	° ′	
1	17 56½ D	37 12	516 D
2	10 12½	41 50	359
3	5 41	44 48	229
4	2 25	47 2	107
5	0 29	49 6	20
6	1 12 E	51 0	74 A
7	3 0	52 35½	198
8	5 0	54 44½	378
9	6 18	55 56½	520
10	7 2	56 47½	620
11	8 52	57 24	821
12	10 10	57 55½	986
13	11 24	58 28	1160
14	12 20½	58 58	1316

N.B. The place of this laſt pole would ſeem to be the ſame as N the Weſtern cairn, as the ſection was directed through it. But then the laſt number 1316 is too great; as, from all the other meaſures, the diff. in alt. between A and N is only 1303 feet. This diff. of 13 feet ſeems to be cauſed by the laſt bearing being about 7′ too great, for in other places this angle is only 58° 51′. And indeed many other angles taken at the ſame time with the above ſeem to be much wrong, as they greatly differ from correſponding ones taken at other times.

Such differences among correſponding angles I often met with in the meaſures contained in the books of the ſurvey, and it required much care to detect them, and trouble to reconcile them.

SECTION 9.

Pole.	Bearings at A.	Bearings at B.
	° ′	° ′
1	132 21½	36 45½
2	130 27	37 34½
3	127 26	39 8
4	124 25½	40 18
5	119 6½	43 19
6	111 37	48 7
7	103 45½	52 58
8	95 25	58 50
9	85 55	67 54
10	78 2½	75 58½
11	73 14	82 30
12	71 41½	86 21
13	70 38½	89 33
14	69 39	92 23

This is a horizontal or level ſection through A, and therefore each point is 4½ feet (the height of the theodolite) above that point.

SECTION 10.

Pole.	Vert. ∠'s at A.	Bearings at B.	Diff. of Alt.
	° ′	° ′	
1	19 41 D	51 12	531 D
2	13 52	57 22½	426
3	8 7½	62 56	295
4	2 30	68 47	106
5	0 59	73 55½	47
6	0 22½ E	76 52	27 A
7	0 59	79 14½	70
8	1 39	81 7	124
9	2 25	82 42	194
10	3 5	84 12	266
11	3 36	85 41½	338
12	3 48	86 27	373

SECTION 11.

Pole.	Vert. ∠'s at D.	Bearings at C.	Diff. of Alt.
	° ′	° ′	
1	12 56 D	48 18	231 D
2	9 50	62 4	334
3	5 15	67 9	242
4	2 25	69 51½	135
5	0 59 E	71 20	70 A
6	3 0	72 13	220
7	4 31	73 9	363
8	6 7	73 53	533
9	6 59	74 27	652
10	7 53	74 52	777
11	8 49	75 9½	904
12	9 27	75 42	1048
13	9 44	76 12½	1168
14	9 53	76 34	1258
15	10 38	76 55½	1443

SECTION 12.

Pole.	Bearings at D.	Bearings at C.	
	° ′	° ′	Each of theſe poles is five feet above D, the ſection being horizontal and taken at that point.
1	68 14½	102 6	
2	71 16	98 23	
3	73 55	94 56	
4	79 0	89 33	
5	85 28	82 49	
6	91 19	76 48	
7	97 5	71 8	
8	102 21	66 20	
9	108 18	60 36	
10	113 33	56 15	
11	117 12	53 17	
12	119 16	52 4	
13	119 53	52 1	

SECTION 13.

Pole.	Vert. ∠'s at G.	Bearings at F.	Diff. of Alt.
	° ′	° ′	
1	3 7 D	75 2	92 D
2	0 12	79 46	2
3	4 1 E	91 55	221 A
4	6 8	95 0	385
5	7 49	96 34	530
6	9 50	99 27	789
7	10 52	100 10	915

	° ′	° ′	
6	3 38	54 6	155
7	1 15	56 50	57
8	0 55 F	60 7	55 A
9	4 4	62 58	257
10	6 45	66 2	487

SECTION 21.

Pole.	Vert. ∠'s at L.	Bearings at W.	Diff. of Alt.
	° ′	° ′	
1	23 59 D	21 18	295 D
2	19 25	33 12	377
3	14 37	48 26	418
4	10 5	56 7	383
5	5 9	62 51	237
6	1 2	69 24	56
7	2 2 E	71 36	133 A
8	4 22	73 2	297
9	6 6	75 22	455

SECTION 22.

Pole.	Vert. ∠'s at *a*.	Bearings at *b*.	Diff. of Alt.
	° ′	° ′	
1	7 16 D	5 2	20 D
2	2 49 E	10 20	25 A
3	3 33	15 13	44
4	4 18	29 34	120
5	7 0	32 15	222
6	9 1	35 3	331
7	10 26	38 52	471
8	12 19	41 46	657

	° ′	° ′	
9	14 40	43 34	874
10	14 45 top of E. cairn.	43 35	874

SECTION 23.

Pole.	Vert. ∠'s at *a*.	Bearings at *b*.	Diff. of Alt.
	° ′	° ′	
1	15 2 D	4 33	34 D
2	0 51	15 15	4
3	0 10	35 37	0
4	3 7 E	41 53	97 A
5	6 43	42 34	209
6	7 36	43 30	244
7	9 50	45 48	342
8	11 35	59 5	664

SECTION 24.

Pole.	Vert. ∠'s at *b*.	Bearings at *a*.	Diff. of Alt.
	° ′		
1	3 27 D	10 29	29 D
2	0 32 E	21 9	12 A
3	2 58	32 57	64
4	4 40	58 53	133
5	5 14	76 22	173
6	6 24	121 5	340
7	7 50	125 12	448
8	8 27	130 23	546
9	8 55	134 57	664
10	9 6	136 15	712

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SECTION 25.

Pole.	Vert. ∠'s at *b*.	Bearings at *a*.	Diff. of Alt.
	° '	° '	
1	15 39 D	8 14	74 D
2	1 47 E	52 24	54 A
3	3 9	63 13	115
4	5 51	70 37	248
5	9 11	81 35	505
6	11 21	85 10	691
7	12 44	87 45	802

SECTION 26.

Pole.	Vert. ∠'s at *b*.	Bearings at *a*.	Diff. of Alt.
	° '	° '	
1	17 53 D	6 35	64 D
2	0 26	48 5	9
3	2 1 A	60 8	93 A
4	3 4	62 45	151
5	4 40	66 15	256
6	6 20	68 29	374
7	8 10	71 57	550

SECTION 27.

Pole.	Vert. ∠'s at *b*.	Bearings at *a*.	Diff. of Alt.
	° '	° '	
1	4 30 E	59 25	280 A
2	3 33	57 12	204
3	1 47	54 11	92
4	0 10 D	50 15	3 D
5	1 22	46 54	47
	° '	° '	
6	2 20	42 15	69
7	4 50	37 30	122
8	18 52	6 12	65

SECTION 28.

Pole	Vert. ∠'s at T.	Bearings at V.	Diff. of Alt.
	° '	° '	
1	10 23 D	16 36	142 D
2	9 26	23 56	195
3	5 23	28 45	136
4	4 14	32 24	120
5	2 42	37 46	96
6	1 34	41 20	62
7	0 35½	45 24	24
8	1 30 E	48 52	89 A
9	3 30	51 4	220
10	4 20	54 22	307
11	4 49	56 11	367
12	5 15	58 41	443
13	6 0	60 4	537
14	6 47	62 4	664
15	7 24	63 33	778
16	8 16	64 46	924
17	8 46	65 44	1030

SECTION 29.

Pole.	Vert. ∠'s at T.	Bearings at V.	Diff. of Alt.
	° '	° '	
1	8 49 D	26 9	182 D
2	6 51	30 7	163

	° ′	° ′	
3	4 27	38 45	140
4	2 55	44 0	106
5	1 30	49 34	62
6	0 10	52 56	3
7	0 42 E	55 55	42 A
8	2 12	58 28	132
9	3 3	60 58	195
10	3 36	63 32	247
11	4 4	66 16	303
12	4 8	68 28	331
13	4 27	70 9	377
14	5 0	71 54	450
15	5 7	74 23	506

SECTION 30.

Pole.	Vert. ∠'s at T.	Bearings at V.	Diff. of Alt.
	° ′	° ′	
1	3 26 E	81 0	353 A
2	3 10	79 53	313
3	2 55	77 7	263
4	2 46	75 2	233
5	2 35	72 45	204
6	2 26	70 16	179
7	1 20	66 28	91
8	0 19	62 32	23
9	0 49 D	57 58	37 D
10	2 33	51 54	107
11	4 23	43 37	151
12	6 21	35 48	176
	° ′	° ′	
13	8 3	29 13	143
14	10 2	21 54	170

SECTION 31.

Pole.	Vert. ∠'s at Y.	Bearings at V.	Diff. of Alt.
	° ′	° ′	
1	11 18 D	10 40	132 D
2	5 29	15 43	93
3	2 56	19 13	60
4	1 42	22 49	40
5	0 42	28 23	20
6	0 16	31 43	5
7	0 30 E	36 10	28 A
8	1 33	40 41	90
9	2 13	45 0	146
10	2 41	47 18	190
11	2 54	50 47	231
12	3 13	53 12	278
13	3 39	56 0	349
14	4 53	58 51	519
15	5 42	60 38	650
16	6 0	62 8	728
17	6 27	63 23	825
18	6 41	64 21	892
19	6 55	65 50	988

SECTION 32.

Pole.	Vert. ∠'s at Y.	Bearings at Y.	Diff. of Alt.
	° ′	° ′	
1	7 9 E	51 46	812 A
2	6 55	50 50	751
3	6 7	49 45	631
4	5 0	46 51	452
5	4 25	45 30	377
6	3 29	43 37	275
7	3 5	40 32	214
8	2 17	38 26	146
9	1 21	35 57	80
10	0 52	33 0	47
11	0 3	29 47	7
12	1 3 D	26 24	33 D
13	2 3	23 40	60
14	3 21	18 30	73
15	7 54	13 4	121

SECTION 33.

Pole.	Vert. ∠'s at T.	Bearings at Z.	Diff. of Alt.
	° ′	° ′	
1	13 6 D	4 6	108 D
2	9 15	8 4	139
3	7 32	14 53	187
4	5 10	21 28	167
5	3 55	26 54	148
6	2 36	36 19	119
7	1 30	45 32	78
8	0 40	56 5	37
	° ′	° ′	
9	0 28 E	65 38	38 A
10	1 40	71 15	132
11	2 22	77 3	197
12	2 25	87 12	226
13	2 32	93 33	257

SECTION 34.

Pole.	Vert. ∠'s at T.	Bearings at Z.	Diff. of Alt.
	° ′	° ′	
1	4 43 E	84 3	471 A
2	4 8	79 45	386
3	3 31	76 0	311
4	2 45	71 30	229
5	2 0	66 22	156
6	1 8	60 49	84
7	0 19	53 20	25
8	0 55 D	46 47	46 D
9	1 29	40 15	69
10	1 59	34 52	83
11	3 41	26 15	127
12	5 27	21 55	165
13	6 0	17 16	150
14	9 5	9 56	143
15	14 6	4 32	109

SECTION 35.

Pole.	Vert. ∠'s at T.	Bearings at Z.	Diff. of Alt.
	° ′	°	
1	15 16 D	1 58	86 D
2	9 36	5 3	126

	° ′	° ′	
3	8 21	8 5	162
4	7 34	12 54	208
5	5 38	17 50	191
6	4 21	22 48	170
7	3 12	32 56	153
8	2 31	41 54	135
9	1 42	54 12	103
10	1 10	64 26	76
11	0 52	75 48	61
12	0 30	86 54	36
13	0 14	94 17	15

SECTION 36.

Pole.	Vert. ∠'s at U.	Bearings at Z.	Diff. of Alt.
	° ′	° ′	
1	17 42 D	2 45	60 D
2	15 55	22 26	465
3	12 39	25 13	404
4	8 51	29 37	333
5	7 23	33 36	318
6	3 51	40 33	205
7	1 22	44 45	80
8	1 3 E	48 26	77 A
9	2 54	50 52	218
10	4 22	52 45	347
11	5 37	54 40	471
12	6 30	55 30	559

SECTION 37.

Pole.	Vert. ∠'s at U.	Bearings at Z.	Diff. of Alt.
	° ′	° ′	
1	3 53 E	63 50	345 A
2	3 27	60 54	286
3	1 55	57 45	150
4	0 16	54 3	23
5	1 14 D	50 40	74 D
6	2 53	47 28	166
7	4 45	43 0	247
8	6 25	38 49	300
9	8 40	33 14	348
10	12 6	28 25	420
11	16 36	23 43	419

SECTION 38.

Pole.	Vert. ∠'s at U.	Bearings at Z.	Diff. of Alt.
	° ′	° ′	
1	15 9 D	20 37	405 D
2	11 30	31 39	438
3	9 23	36 7	398
4	6 46	40 30	314
5	5 27	46 10	283
6	3 32	52 5	204
7	2 49	55 9	171
8	1 49	61 25	122

SECTION 39.

Pole.	Vert. ∠'s at U.	Bearings at Z.	Diff. of Alt.
	° ′	° ′	
1	4 28 D	69 7	312 D
2	5 35	59 8	345
3	6 30	52 3	366
4	8 9	45 25	418
5	10 32	38 30	485
6	12 47	34 3	544
7	13 47	29 27	531

SECTION 40.

Pole.	Vert. ∠'s at P.	Bearings at P.	Diff. of Alt.
	° ′	° ′	
1	8 45 D	9 15	735 D
2	8 45	16 25	750
3	9 58	27 16	906
4	8 38	30 52	804
5	7 6	34 30	681
6	5 23	37 55	533

SECTION 41.

Pole.	Vert. ∠'s at O.	Bearings at A.	Diff. of Alt.
	° ′	° ′	
1	10 41 D	23 33	510 D
2	11 31	32 36	769
3	8 52	37 23	682
4	5 38	44 24	524
5	4 52	46 39	480
6	4 6	51 8	455
	° ′	° ′	
7	2 8	56 26	269
8	1 21	58 30	178
9	0 57	59 53	129
10	0 43	62 21	103
11	0 40	64 35	102
12	0 5	68 41	10
13	0 10 E	69 43	36 A
14	0 32	71 20	109

SECTION 42.

Pole.	Vert. ∠'s at M.	Bearings at S.	Diff. of Alt.
	° ′	° ′	
1	0 17 D	73 48	0
2	3 25 E	80 57	72 A
3	4 52	85 4	126
4	6 52	88 53	232
5	8 33	90 25	332
6	10 2	91 34	439
7	11 44	93 0	610
8	13 7	94 0	787
9	15 10	94 34	1002
10	17 21	95 13	1292
11	18 38	95 37	1506
12	19 36	96 2	1736
13	19 38	96 0	1727

SECTION 43.

Pole.	Vert. ∠'s at O.	Bearings at A.	Diff. of Alt.
	° ′	° ′	
1	12 3 D	31 52	759 D
2	10 54	36 42	776

	° ′	° ′	
3	8 58	41 8	705
4	7 2	46 9	614
5	5 35	49 34	521
6	3 33	58 6	389
7	2 52	62 38	341
8	2 14	67 20	290
9	2 12	85 34	421

SECTION 44.

Pole.	Vert. ∠'s at R.	Bearings at S.	Diff. of Alt.
	° ′	° ′	
1	3 41 D	6 37	32 D
2	1 18	10 10	21
3	0 51 E	11 21	25 A
4	0 52	12 20	30
5	1 4	16 25	70
6	2 3	16 52	143
7	3 6	17 27	243
8	4 38	18 0	413
9	5 14	18 6	478
10	5 41	18 11	530
11	6 28	18 27	647

SECTION 45.

Pole.	Vert. ∠'s at A.	Bearings at R.	Diff. of Alt.
	° ′	° ′	
1	2 15 D	30 14	122 D
2	2 0	31 57	116
3	1 22	38 35	100
	° ′	° ′	
4	0 8 E	42 19	16 A
5	4 38	49 48	517
6	5 12	50 45	598
7	5 58	52 13	721
8	6 37	54 58	878
9	7 18	56 7	1008
10	7 22	56 39	1037

SECTION 46.

Pole.	Vert. ∠'s at D.	Bearings at C.	Diff. of Alt.
	° ′	° ′	
1	8 50 D	96 20	201 D
2	7 24	111 4	260
3	6 36	114 46	272
4	6 9	116 22	275
5	5 24	119 44	298
6	5 4	121 17	315
7	4 16	122 24	292
8	3 27	123 13	255
9	2 23	124 17	197
10	1 17	124 55	113
11	0 44 E	125 50	82 A
12	1 17	126 8	146

SECTION 47.

Pole.	Vert. ∠'s at T.	Bearings at V.	Diff. of Alt.
	° ′	° ′	
1	10 42 D	17 45	152 D
2	6 6	28 48	145
3	3 19	38 30	111

	° ′	° ′	
4	2 4	43 5	79
5	0 44	47 48	30
6	0 30 E	49 46	30 A
7	2 28	52 45	143
8	3 24	54 21	205
9	4 39	59 0	335
10	4 54	62 16	387
11	5 57	64 50	517
12	6 16	65 38	562
13	6 50	67 39	666
14	7 11	69 40	764

SECTION 48.

Pole.	Vert. ∠'s at V.	Bearings at Y.	Diff. of Alt.
	° ′	° ′	
1	9 0 D	17 3	164 D
2	7 0	30 56	236
3	3 22	44 36	170
4	1 16	47 23	66
5	0 5	49 52	0
6	2 46 E	58 6	214 A
7	4 57	64 34	457
8	5 53	67 2	585
9	6 21	70 8	696
10	6 30	72 34	772
11	6 34	75 13	857
12	7 41	77 32	1097
13	8 39	79 6	1316
14	8 46	79 34	1361
	° ′	° ′	
15	8 52	80 32	1435
16	9 6	82 53	1641

SECTION 49.

Pole.	Vert. ∠'s at T.	Bearings at e.	Diff. of Alt.
	° ′	° ′	
1	11 56 D	16 58	83 D
2	9 12	35 25	120
3	8 14	64 20	190
4	8 12	80 0	250
5	5 55	92 52	234
6	4 14	98 36	193
7	1 58	103 12	101
8	0 34 E	106 12	39
9	1 20	108 26	92
10	2 15	109 6	157
11	3 7	110 35	230
12	3 49½	111 48	300

SECTION 50.

Pole.	Vert. ∠'s at L.	Bearings at W.	Diff. of Alt.
	° ′	° ′	
1	17 46 D	10 52	226 D
2	17 2	13 33	296
3	14 51	16 2	333
4	10 47	20 13	363
5	9 26	23 0	418
6	8 54	24 37	466
7	8 7	26 16	508

	° ′	° ′	
8	7 8	27 23	507
9	6 17	28 27	507
10	4 55	29 28	451
11	4 30	30 12	490
12	3 51	30 55	433
13	3 2	31 15	357
14	2 35	31 29	315

SECTION 51.

Pole.	Vert. ∠'s at F.	Bearings at F′.	Diff. of Alt.
	° ′	° ′	
1	14 37 D	3 57	73 D
2	2 56	44 8	140
3	1 43	48 47	90
4	1 3	55 5	61
5	1 0	60 58	65
6	0 41	67 26	50
7	0 23	74 28	31
8	0 25 E	82 0	52 A
9	1 53	83 46	225
10	3 17	87 0	423
11	3 32	87 28	461

SECTION 52.

Pole.	Vert. ∠'s at F′.	Bearings at F.	Diff. of Alt.
	° ′	° ′	
1	2 48 E	68 41	187 A
2	3 50	83 0	282
3	6 45	102 0	630
	° ′	° ′	
4	7 6	108 57	728
5	8 4	111 38	863
6	8 37	112 58	943
7	9 19	115 56	1075
8	10 8	118 30	1230
9	10 57	120 40	1393
10	11 30	122 30	1528
11	12 22	124 50	1748
12	13 40	127 12	2075
13	14 3	128 13	2205
14	14 7	128 32	2239

SECTION 53.

Pole.	Vert. ∠'s at F′.	Bearings at F.	Diff. of Alt.
	° ′	° ′	
1	3 11 E	64 3	208 A
2	5 9	71 24	366
3	5 58	75 50	448
4	6 14	84 12	522
5	8 51	91 23	821
6	9 0	97 47	924
7	10 7	99 40	1074
8	11 2	102 33	1236
9	11 45	104 44	1375
10	12 6	106 28	1469
11	12 43	108 24	1612
12	12 55	109 18	1673

SECTION 54.

Pole.	Vert. ∠'s at t′.	Bearings at F′.	Diff. of Alt.
	° ′	° ′	
1	0 22 E	14 7	12 A
2	1 27	20 5	44
3	5 13	39 2	241
4	5 46	45 18	299
5.	6 9.	55 40	379
6	6 38	62 0	451
7	7 11	70 8	553
8	7 30	78 56	662
9	7 48	84 30	755
10	7 50	94 8	908
11	7 54	98 26	1006
12	7 58	100 58	1077

SECTION 55.

Pole.	Vert. ∠'s at t′.	Bearings at F′.	Diff. of Alt.
	° ′	° ′	
1	2 0 E	11 46	35 A
2	2 3	22 45	62
3	4 3	25 32	130
4	7 32	34 49	324
5	9 15	42 55	497
6	9 52	48 8	604
7	10 30	53 40	736
8	10 33	62 34	917
9	11 7	66 12	1060
10	11 57	69 15	1235
11	11 52	73 19	1370

SECTION 56.

Pole.	Vert. ∠'s at t′.	Bearings at F′.	Diff. of Alt.
	° ′	° ′	
1	3 11 E	9 4	41 A
2	4 27	16 21	97
3	7 15	21 33	205
4	9 12	27 0	331
5	10 30	29 15	414
6	11 18	31 14	480
7	11 26	35 39	569
8	12 21	38 50	686
9	12 42	41 44	777
10	13 18	44 21	887
11	13 32	46 36	971

SECTION 57.

Pole.	Vert. ∠'s at k.	Bearings at t′.	Diff. of Alt.
	° ′	° ′	
1	7 51 E	29 43	176 A
2	11 24	34 13	299
3	13 22	37 54	397
4	14 52	42 20	509
5	16 40	47 44	675
6	16 46	50 15	833

SECTION 58.

Pole.	Vert. ∠'s at k.	Bearings at t′.	Diff. of Alt.
	° ′	° ′	
1	0 3 E	5 28	5 A
2	6 10	8 5	44

	° ′	° ′	
3	10 3	13 52	121
4	10 20	29 37	313
5	14 20	32 18	497
6	15 0	33 18	547

SECTION 59.

Pole.	Vert. ∠'s at M'.	Bearings at h.	Diff. of Alt.
	° ′	° ′	
1	28 16 D	Not ſeen.	
2	25 20	66 56	910 D
3	22 23	74 50	1056
4	22 12	76 20	1114
5	21 40	79 10	1232
6	21 20	83 32	1521
7	20 14	85 50	1656
8	19 12	87 30	1758
9	18 46	89 17	1978
10	17 30	90 20	2016
11	16 8	91 5	1990
12	14 42	92 27	2093
13	13 35	92 54	2035
14	12 46	93 14	1991
15	11 30	93 45	1915
16	10 30	94 7	1845

The following are the irregular ſections. In the firſt column is the number of poles; in the ſecond the vertical angles; in the third and fourth the two bearings or horizontal angles at each end of the baſe; and in the fifth the computed reſult, being the difference of altitude between the foot of each pole and the point mentioned in the ſecond column where the vertical angles were taken.

SECTION 60.

Pole.	Vert. ∠'s at H.	Bear. at H.	Bear. at G.	Diff. of Alt.
	° ′	° ′	° ′	
1	8 30E	70 55	66 39	832A
2	8 30	66 46	70 33	850
3	8 30	62 56	75 9	884
4	8 30	58 20	79 20	892
5	8 30	54 27	82 42	891

SECTION 61.

Pole.	Vert. ∠'s at H.	Bear. at H.	Bear. at G.	Diff. of Alt.
	° ′	° ′	° ′	
1	9 31E	68 51	71 0	1004A
2	9 31	64 19	77 50	1091
3	9 31	60 19	80 38	1072
4	9 31	55 40	84 34	1066

SECTION 62.

Pole.	Vert. ∠'s at H.	Bear. at H.	Bear. at G.	Diff. of Alt.
	° '	° '	° '	
1	11 0E	72 45	72 40	1334A
2	11 0	69 57	76 21	1389
3	11 0	62 40	82 29	1376
4	11 0	59 7	84 22	1327

SECTION 63.

Pole.	Vert. ∠'s at H.	Bear. at H.	Bear. at G.	Diff. of Alt.
	° '	° '	° '	
1	12 15E	74 9	74 0	1613A
2	12 15	70 14	78 7	1652
3	12 15	67 32	80 52	1669
4	12 15	64 2	84 1	1664

SECTION 64.

Pole.	Vert. ∠'s at W.	Bear. at W.	Bear. at L.	Diff. of Alt.
	° '	° '	° '	
1	5 57E	74 37	81 12	382A
2	6 57	78 12	76 36	333
3	7 11	81 12	73 19	329
4	6 19	86 33	68 34	394
5	7 5	92 16	64 4	334
6	7 48	96 43	60 49	271
7	8 40	100 10	58 40	184
8	8 52	102 42	56 41	167
9	8 41	106 0	54 23	168
10	8 30	108 53	52 16	176

SECTION 65.

Pole.	Vert. ∠'s at b.	Bear. at b.	Bear. at a.	Diff. of Alt.
	° '	° '	° '	
1	2 1D	47 47	19 56	20D
2	1 15E	32 1	26 45	26A
3	2 59	27 0	32 56	65
4	4 35	23 40	45 18	118
5	5 0	19 2	63 47	150
6	5 39	15 26	98 12	202
7	6 1	13 3	121 47	238
8	5 44	9 29	143 12	247

SECTION 66.

Pole.	Vert. ∠'s at d.	Bear. at d.	Bear. at c.	Diff. of Alt.
	° '	° '	° '	
1	14 25D	162 37	11 30	870D
2	15 55	158 15	13 36	823
3	16 57	152 52	16 35	827
4	18 12	149 43	18 0	831
5	20 57	147 3	18 41	866
6	23 5	136 18	23 37	865
7	23 40	129 0	27 23	876
8	23 3	122 16	31 32	877
9	23 46	112 59	36 25	894
10	23 23	104 22	41 34	891
11	23 35	91 13	48 53	892
12	21 21	82 49	57 38	902
13	20 11	77 33	63 0	897
14	18 47	69 33	71 24	890
15	17 31	65 26	77 38	893

	° ′	° ′	° ′	
16	16 1	61 51	83 57	884
17	14 33	59 1	89 55	875
18	12 48	56 49	96 15	868
19	11 38	54 45	100 45	850

SECTION 67.

Pole.	Vert. ∠'s at P.	Bear. at P.	Bear. at H.	Diff. of Alt.
	° ′	° ′	° ′	
1	5 42D	47 54	61 12	476D
2	7 42	54 10	53 56	592
3	7 13	55 23	52 22	542
4	7 12	58 40	50 36	532
5	8 1	63 52	46 43	564
6	6 47	68 29	44 48	469
7	5 27	71 25	43 18	369
8	5 31	79 14	40 16	368
9	4 15	81 17	39 11	278
10	3 51	87 37	36 34	247
11	3 48	90 58	35 16	242
12	1 5	96 47	32 56	64
13	1 27	96 53	30 46	80
14	3 42	91 16	32 0	208
15	6 0	83 54	34 35	348
16	7 21	69 39	38 26	435
17	9 5	64 1	41 18	564
18	9 34	55 56	44 56	625
19	10 1	48 45	49 59	706
20	9 24	41 45	54 41	703
21	9 52	37 19	59 16	777
	° ′	° ′	° ′	
22	10 0	32 45	65 51	841
23	9 22	29 27	71 0	819

SECTION 68.

Pole	Vert. ∠'s at P.	Bear. at P.	Bear. at G.	Diff. of Alt.
	° ′	° ′	° ′	
1	6 23D	21 51		
2	6 20	22 44	139 42	140D
3	6 20	24 2	136 17	1092
4	6 14	25 20	132 35	1025
5	5 54	27 20	127 52	932
6	5 41	29 23	121 57	843
7	5 21	32 27	115 58	769
8	5 29	35 59	107 50	740
9	5 45	41 31	87 8	615
10	5 30	45 10	82 44	578
11	5 42	49 50	77 53	589
12	5 56	53 57	74 41	613
13	6 3	58 6	70 2	605
14	5 4	64 0	66 29	510
15	5 8	67 27	65 5	527
16	4 11	73 46	65 27	486
17	4 20	76 52	64 11	518
18	4 21	80 26	62 58	542

SECTION 69.

Pole.	Vert. ∠'s at H.	Bear. at H.	Bear. at W.	Diff. of Alt.
	° ′	° ′	° ′	
1	12 47E	116 58	51 34	1809A
2	14 36	107 42	59 25	2135

3	13° 56′	101° 11′	65° 9′	2023	8	12° 3′	78° 24′	85° 45′	1656
4	13 43	96 35	69 7	1960	9	11 47	76 54	87 30	1646
5	13 9	92 53	72 30	1875	10	11 26	74 56	89 25	1592
6	12 38	88 41	76 6	1760	11	10 6	68 32	96 23	1452
7	12 28	82 53	81 18	1703					

The three following ſections were taken in a manner different from all the reſt. They were made by meaſuring in a ſtreight ſloping line (or nearly ſtreight) from certain points towards K and N, and at the beginning of the line taking the angle of elevation or depreſſion of ſeveral places or points in it, whoſe diſtance from the beginning were meaſured. In theſe caſes each diſtance is the hypothenuſe of a right-angled triangle, and the manner of operation is this, as radius is to the hypothenuſe or meaſured ſlope diſtance, ſo is the ſine of the elevation or depreſſion to the difference of altitude, and ſo is the coſine of the ſame vertical angle to the horizontal diſtance.

SECTION 70, from M′ to K.

Pole.	Slope Diſt.	Vert. ∠'s at M′.	Horiz. Diſt.	Diff. of Alt.
1	463	7° 50½′ D	459	60
2	794	6 54½	788	92
3	992	6 50	985	114
4	1257	5° 28½′	1251	116
5	1455	5 25½	1449	134
6	1824 Ends at K.	5 4	1817	157

SECTION 71, from δ′ to K.				
Pole.	Slope Diſt.	Vert. ∠'s at δ.	Horiz. Diſt.	Diff. of Alt.
1	727	13° 39½′ D	706	167
2	1455	9 44½	1434	242
3	1720	9 4½	1699	267
4	1984	7 52	1965	267
5	2547 Ends at K.	7 3	2528	308

SECTION 72, from δ′ to N.				
Pole.	Slope Diſt.	Vert. ∠'s at δ.	Horiz. Diſt.	Diff. of Alt.
1	528	3° 10′ E	527	38
2	1188	4 10	1185	95
3	1594 Ends at N.	6 8	1585	179

Beſides theſe ſections there were many more ſingle points, whoſe places and relative altitudes were obſerved and computed; but it is not neceſſary to abſtract them all here.

The following plate (Tab. VIII.) has 72 figures anſwering to theſe 72 ſections, each to each, according to the numbers. In theſe figures, the line having the letters p, p, p, &c. annexed is the ſection line; the letters p, p, &c. denoting the poles; the other line, forming the angle with the ſection, is the baſe line; and between them are the degrees and minutes contained in the angle formed by them; at the angular point was obſerved the elevation or depreſſion of each point p, and the bearings or horizontal angles were obſerved at the other end of the baſe, from whence faint lines are drawn to ſome of the points p forming with the baſe line thoſe horizontal angles. The baſe and ſection lines in each figure are alſo drawn nearly in the ſame direction as they are in the plan or on the ground, ſuppoſing the top of the paper to be the North, towards which a perſon looks when viewing the ground from the South.

Having finiſhed the computation of the relative altitudes of all the points, the next conſideration is how they are to be applied in determining the attraction of the hill. In whatever manner this laſt mentioned operation may be performed, it is evident, that all the points and ſections with their altitudes muſt be entered in the plan. Wherefore, having accurately conſtructed a large plan of the ground, as before mentioned, containing all the principal lines or baſes, at the extremities of which either vertical or horizontal angles were taken, from them I then determined in this plan the places of all the other points in the ſections, whether vertical, horizontal, or irregular. Theſe places or points were determined by drawing lines from each extremity of the baſe ſo as to form with it angles equal to thoſe which were obſerved on the ground for each correſponding pole; the interſections of theſe lines are the places of the poles, which having marked with a fine dot or point of ink, and written cloſe to each point the proper number expreſſing its relative altitude, and cleaned the paper by rubbing out the lines forming the angles by which the points were determined, there remained only the points with the figures expreſſing their altitudes diſtinctly exhibited in the plan (ſee tab. IX.)

It remains now to apply all the foregoing calculations and conſtructions to the determination of the effect of the attraction in the direction of the meridian. And here it ſoon occurred, that the beſt method was to divide the plan into a great number of ſmall parts, which may be conſidered as the baſes of as many vertical columns or pillars of matter into which the hill and the adjacent ground may be ſuppoſed to be divided by vertical planes, forming an imaginary group of vertical columns, ſomething like a ſet of baſaltine pillars, or like the cells in a piece of honey-comb; then to compute the attraction of each pillar ſeparately in the direction of the meridian; and laſtly, to take the ſum of all theſe computed effects for the whole attraction of the matter in the hill, &c. Now the attraction of any one of theſe pillars on a body in a given place may be eaſily determined, and that in any direction, to a ſufficient degree of accuracy, becauſe of the ſmallneſs and given poſition of the baſe; for, on account of its ſmallneſs, all the matter in the pillar may be ſuppoſed to be collected into its axis or vertical line erected on the middle of the baſe, the length of which axis, as the mean altitude of the pillar, is to be eſtimated from the altitudes of the points in the plan which fall within and near the baſe of the pillar: then, having given the altitude of this axis, with the poſition of its baſe, and the matter ſuppoſed to be collected into it, a theorem can eaſily be given by which the effect of its attraction may be computed. But to retain the proper degree of accuracy in this computation, it is evident that the plan muſt be divided into a great number of parts, perhaps not leſs than a thouſand for each obſervatory, in order that they may be ſufficiently ſmall, and by this means forming about two thouſand of ſuch pillars of matter, whoſe attractions muſt be ſeparately computed, as mentioned above. The labour and time neceſſary for ſuch computation, it is evident, would be very great, perhaps not leſs than thoſe employed in all the preceding computations of the ſections, and all the other points and lines concerned in this buſineſs. For this reaſon I was deſirous of obtaining a theorem or method by which the attractions of the ſmall and numerous pillars might be computed with the ſame degree of accuracy, but with leſs expence of labour and time than when computed ſeparately as above mentioned. And in this inquiry the ſucceſs has been equal to my wiſhes, having at length met with a method by which the buſineſs has been effected in perhaps one-fourth or one-fifth of the time that would have been required in the other way. This method I have inveſtigated partly from ſome hints of the honourable HENRY CAVENDISH, F. R. S. and partly from ſome of my own, which had been communicated to the Aſtronomer Royal in the years 1774 and 1775: of which method and its inveſtigation I ſhall now give ſome account.

Of all the methods of dividing the plan into a great number of ſmall parts, I have found that to be the moſt convenient for the computation, in which it is firſt divided into a number of rings by concentric circles, and theſe again divided into a ſufficient number of parts by radii drawn from the common center, that center being the obſervatory where the plummet is placed on which the effect of attraction is to be computed. By this means the plan is divided into a great number of ſmall quadrilateral ſpaces, two of the oppoſite ſides of which are ſmall portions of adjacent circles, and the other two are the intercepted ſmall parts of two adjacent radii, as appears by fig. 1. tab. X. in which, for the preſent, let the circles and their radii be ſuppoſed to be drawn at any diſtances whatever from each other, till it ſhall appear from the theorem to be inveſtigated what may be the propereſt diſtances and poſitions of thoſe lines. In this figure A is the obſervatory, AN the meridian, NAE an

Eaſt-and-Weſt line, BCDE one of the little ſpaces, and F its center or foot of the axis of the pillar whoſe baſe is BCDE; the figure AWNEA being a horizontal or level ſection through the point A. Join A, F, and with the center A deſcribe the middle circle GFH. Let a denote the length of the axis on the point F, or the mean height of the pillar on the baſe BD; and $s =$ the ſine of the angle of elevation of that pillar as obſerved at A, to the radius 1, or $s = \frac{a}{\sqrt{a^2 + \mathrm{AF}^2}}$. Then will the magnitude of that column or its quantity of matter be expreſſed by $\frac{\mathrm{BC}+\mathrm{ED}}{2} \times \mathrm{BE} \times a$, which is ſuppoſed to be all collected into the axis: conſequently, if the attraction of each particle of matter be in the reciprocal duplicate ratio of its diſtance, the attraction of the matter in the pillar, ſo placed on the plummet at A, in the direction of the meridian AN, will be $\frac{\mathrm{BC}+\mathrm{ED}}{2\mathrm{AF}} \times \mathrm{BE} \times a \times \frac{s}{a} \times c = \frac{\mathrm{BC}+\mathrm{ED}}{2\mathrm{AF}} \times \mathrm{BE} \times sc = \frac{\mathrm{GH}}{\mathrm{AF}} \times \mathrm{BE} \times sc$ nearly, ſuppoſing F to be equally diſtant from BC and ED; and c the coſine of the angle FAN to the radius 1.

But $\frac{\mathrm{GH}}{\mathrm{AF}} \times c$ is nearly equal to d the difference of the ſines of the angles BAN, CAN, as is thus demonſtrated. Draw GK, FL, HM, perpendicular, and GP parallel to AW; and draw the chord GH. Then AK, AM are the ſines of the angles GAN, HAN, to the radius AF, their difference being KM = GP; alſo FL is the coſine of FAN to the ſame radius: conſequently GP : FL = $d : c$. But the triangles LFA, PGH are equiangular, and therefore GP : FL = GH : AF. Conſequently GH : AF = $d : c$; or $\frac{GH}{AF} \times c = d$. This equation is accurately true when GH is the chord of the arc; and as the ſmall arc differs inſenſibly from its chord, the ſame equation is ſufficiently near the truth when GH is the arc itſelf. Subſtituting now d inſtead of the quantity $\frac{GH}{AF} \times c$ in the theorem above, it will become $BE \times ds$ for the meaſure of the attraction of the pillar whoſe baſe is BD in the direction AN. Which is as eaſy and ſimple an expreſſion for the attraction of a ſingle pillar as can well be deſired or expected.

But to make the application of this theorem ſtill more eaſy to the great number of ſmall pillars concerned in this buſineſs, let us ſuppoſe BE and d to be conſtant or invariable quantities, and then it is evident that we ſhall have nothing more to do but to collect all the s's or ſines of elevation of all the pillars into one ſum, and then multiply that ſum by the conſtant quantity $BE \times d$, by which there will be produced the meaſure of the attraction of all the pillars, or of the whole part of the ground on one ſide of WE. Now BE will be made to become conſtant, by making the circles equi-diſtant from one another, or by taking the radii in arithmetical progreſſion. And *d* will be conſtant, by drawing the radii ſo as to form with AN angles whoſe ſines ſhall be in arithmetical progreſſion; for then *d* is the common difference of the ſines of thoſe angles. Hence then we are eaſily led to the beſt manner of dividing the plan into the ſmall ſpaces, *viz.* from the center A deſcribe a ſufficient number of concentric and equi-diſtant circles; divide the radius AI of any one of them into a ſufficient number of equal parts, and from the points of diviſion erect perpendiculars to meet the circle; then through the points of interſection draw radii, and they will divide the circles in the manner required.

In a computation of this kind, we need only calculate the attraction of the matter above the plane or horizon of each obſervatory, and the attraction of ſo much matter as is wanting to fill up the vacuity below that plane lying between it and the ſurface of the lower part of the hill. For the South obſervatory, the attraction of the Southern parts that are above it muſt be ſubtracted from that of the Northern parts, to obtain the attraction of the whole towards the North; that is, the Southern elevations are negative, and the Northern ones affirmative. The contrary names take place with reſpect to the depreſſions, or the vacuities below the plane of the obſervatory; for if the whole ſpace below this horizontal plane were full of matter to an equal extent both ways, its attraction need not be computed, as thoſe on the contrary ſides would mutually balance each other; but ſince there are unequal vacuities on each ſide, it is evident, that the attraction of the matter that might be contained in them muſt be deducted from the other two equal quantities, to leave the real attraction of thoſe two ſides; then ſubtracting the remainder to the South ſide from that of the Northern ſide, there will at laſt remain the joint effect of all the matter below the plane in the Northern direction: but as the one remainder is to be ſubtracted from the other, the two equal quantities may be omitted in both, and only the effects of the vacuities brought into the account, which being twice ſubtracted, their ſigns become contrary to thoſe of the parts above the horizontal plane; that is, the effect of the Southern vacuity is affirmative, and that of the Northern one negative. But for the Northern obſervatory, when the attraction towards the South is to be found, the contrary names take place; that is, in the elevations the Southern parts are affirmative, and the Northern parts negative; but in the vacuities or depreſſions, the Northern parts are affirmative, and the Southern ones negative.

According to the foregoing method the plan of the ground was divided into 20 rings by equidiſtant concentric circles, deſcribed about each obſervatory as a center; and each quadrant was divided into 12 parts or ſectors by lines forming, with the meridian, angles whoſe ſines are in arithmetical progreſſion; by which means the ſpace in each quadrant was divided into 240 ſmall parts, making almoſt a thouſand of ſuch parts in the whole round for each obſervatory, or near 2000 for the two obſervatories. This was judged to be a ſufficiently great number of parts to afford a very conſiderable degree of accuracy; or at leaſt that number was as great, and the parts as ſmall, as was well conſiſtent with the degree of accuracy afforded by the number of points whoſe relative altitudes had been determined.

In this diviſion the common breadth of the rings, or the common difference of the radii, is 666$\frac{2}{3}$ feet; and the common difference of the ſines of the angles formed by the radii and the meridian is $\frac{1}{12}$th of the radius; and conſequently, thoſe angles are expreſſed in degrees and minutes as here follows, *viz.* 4° 47′, 9° 36′, 14° 29′, 19° 28′, 24° 37′, 30° 0′, 35° 41′, 41° 48′$\frac{1}{2}$, 48° 35′, 56° 26′$\frac{1}{2}$, 66° 26′$\frac{1}{2}$, 90° 0′.

Tab. IX. contains a ſmall plan of the principal and moſt central part of the ground, accurately divided in the above manner for one of the obſervatories, namely, the Northern one, with the places of all or moſt of the points which fall within this part of the ground, accurately laid down and marked with dots, as alſo ſuch of the included letters as have been before mentioned in this paper.

In this plate RABCD, &c. is the chain of ſtations around the hill; N and K are the Weſt and Eaſt cairns on the extremities of the ridge of the hill; O the Southern obſervatory, and P the Northern one. Of this kind were made two large plans, one divided for each obſervatory, from which were eſtimated the mean altitudes of the pillars erected on the ſpaces into which they are divided.

Theſe altitudes are eaſily eſtimated when ſeveral of the points fall near and in the ſmall ſpaces or baſes, eſpecially when they are near the middle of them; but, numerous as the points are, there are evidently many baſes in which none at all are contained, nor even near them. This circumſtance at firſt gave me much trouble and diſſatisfaction, till I fell upon the following method by which the defect was in a great meaſure ſupplied, and by which I was enabled to proceed in the eſtimation of the altitudes both with much expedition and a conſiderable degree of accuracy. This method was the connecting together by a faint line all the points which were of the ſame relative altitude: by ſo doing, I obtained a great number of irregular polygons lying within, and at ſome diſtance from, one another, and bearing a conſiderable degree of reſemblance to each other: theſe polygons were the figures of ſo many level or horizontal ſections of the hills, the relative altitudes of all the parts of them being known; and as every baſe or little ſpace had ſeveral of them paſſing through it, I was thereby able to determine the altitude belonging to each ſpace with much eaſe and accuracy. In this eſtimation I could generally be pretty ſure of the altitude to within ten feet, and often within five, which on an average might be about the 100th part of the whole altitude; and when we conſider that the number of ſuch eſtimated altitudes is very great, and that it is probable the ſmall errors among them would nearly balance one another, the defect of thoſe that might be reckoned too little being compenſated by the exceſs in thoſe which might be taken too great, we need not heſitate to pronounce, that the error ariſing from the eſtimation of the altitudes is probably ſtill much leſs than that part.

It was neceſſary to determine theſe altitudes of the pillars, in order to compute the ſines of the angles of elevation ſubtended by them, as the theorem requires the uſe of theſe ſines; and the very eaſy method uſed in deducing the latter from the former ſhall be explained after we have, as below, regiſtered the altitudes of all the pillars as they were computed. This regiſter conſiſts of ſixteen tables, namely four quadrants of ſpaces in the altitudes, and four in the depreſſions, for each obſervatory, as ſpecified in the titles of them. The numbers are feet, like all the other dimenſions. The numbers on the ſame horizontal line from left to right are ſuch as are all in the ſame ring; and thoſe in one and the ſame vertical column are in the ſame ſector, or between the ſame two radii; the number of the ring, counted from the common center, is written in the left-hand margin; and the number of the vertical column or diſtance of the ſpace or ſector from the meridian, at the top; alſo the radius of each ring, that is, the line from the common center to the middle of the ring is written on the ſame line with it, in the right-hand margin. It may be further remarked, that in ſuch little ſpaces as were cut through by the boundary line between elevations and depreſſions, thereby making but a part of ſuch ſpaces in each of thoſe denominations, each ſpace was accounted as a whole one; but then the mean altitude or depreſſion in each part was diminiſhed in the proportion of the whole ſpace to the part of it ſo included in the boundary. The altitudes and depreſſions are put down firſt with reſpect to the Southern obſervatory o, and then for the Northern obſervatory P; and in each, the altitudes are placed firſt.

1. Altitudes above o in the N.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	215	215	215	215	210	205	200	190	170	145	120	75	333⅓
2	605	610	605	600	595	590	580	570	510	450	350	200	1000
3	965	1005	1010	1010	1020	1050	1040	900	810	600	415	220	1667
4	670	680	700	780	800	930	1040	1090	1100	760	480	210	2333
5	280	310	370	450	560	700	830	960	1180	890	545	200	3000
6	20	50	100	110	250	380	525	710	890	950	605	110	3667
7					10	70	120	215	620	780	600	120	4333
8							15	95	390	610	480	35	5000
9									135	310	220	5	5667
10										40	20		6333

2. Altitudes above o in the N.E. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	210	205	205	200	195	185	170	155	140	125	105	70	333⅓
2	550	545	540	530	520	510	500	465	430	370	270	130	1000
3	910	840	825	815	800	760	720	680	635	590	500	200	1667
4	645	640	635	640	645	650	675	715	730	700	580	300	2333
5	265	255	265	285	310	350	390	450	460	500	600	280	3000
6	10	12	20	65	100	130	160	180	180	320	460	300	3667
7								15	55	110	150	250	4333
8												50	5000

3. Altitudes above o in the S.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii
1												10	333⅓
2												15	1000
3												5	1667
12												40	7667
13												60	8333
14												100	9000
15												200	9667
16	40	120	00	250	280	280	260	170	60			300	10333
17	160	270	360	400	440	450	450	390	270	30		500	11000
18	310	420	500	580	620	650	660	630	500	200		700	11667
19	440	540	620	680	740	800	800	780	600	400	60	800	12333
20	550	650	750	800	900	950	960	960	800	500	200	900	13000

4. Altitudes above o in the S.E. quarter.

Rings.	1	2	3	4	5	6	7	8		Radii.
16	10									10333
17	80	60	50	50	10					11000
18	220	200	180	130	70	30				11667
19	340	300	260	240	[illegible]	120	20			12333
20	450	410	380	330	260	180	100	20		13000

5. Depreſſions below o in the N.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
6	70	40	15	5								15	3667
7	250	240	200	150	60	30						40	4333
8	460	450	430	390	280	200	80	10				80	5000
9	700	700	680	630	520	450	340	170	15			120	5667
10	840	830	800	780	650	600	520	380	180	40	70	220	6333
11	960	920	880	850	750	650	630	550	350	250	300	430	7000
12	1100	1000	950	900	820	780	780	780	580	530	500	560	7667
13	1130	1080	980	880	840	800	830	860	780	690	630	640	8333
14	1180	1100	1000	900	900	900	910	940	870	800	700	500	9000
15	1180	1100	1100	1080	1040	1050	1060	1070	1000	870	730	300	9667
16	1100	1100	1100	1100	1100	1140	1150	1150	1120	990	760	160	10333
17	1100	1100	1100	1130	1180	1200	1200	1200	1180	1080	700	80	11000
18	1100	1100	1150	1200	1200	1150	1100	1100	1200	1180	700	100	11667
19	1100	1120	1220	1230	1260	1200	1200	1200	1300	1240	620	60	12333
20	1120	1220	1320	1360	1390	1390	1390	1340	1440	1300	620	50	13000

6. Depreſſions below o in the N.E. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
6	70	60	30	10								10	3667
7	260	240	200	150	110	80	30	40	30	10		10	4333
8	450	440	400	350	280	180	100	180	180	190	190	40	5000
9	700	690	680	610	520	400	200	240	290	350	330	200	5667
10	850	870	890	860	770	620	440	300	380	500	450	370	6333
11	1020	1060	1070	1050	980	800	700	520	400	650	600	530	7000
12	1140	1160	1180	1160	1140	1080	950	840	620	720	850	700	7667
13	1200	1190	1200	1220	1240	1250	1160	1050	900	840	950	880	8333
14	1230	1130	1050	1050	1100	1220	1260	1220	1070	950	1020	990	9000
15	1100	960	900	850	900	1100	1230	1210	1170	1060	1090	1100	9667
16	970	860	880	780	780	900	1120	1180	1200	1180	1160	1150	10333
17	970	800	760	750	750	780	1000	1200	1300	1240	1200	1100	11000

7. Depreſſions below o in the S.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	165	165	160	155	150	140	130	120	110	90	50	10	333
2	400	390	380	370	350	330	300	270	240	210	160	60	1000
3	600	580	560	530	500	570	540	400	370	340	280	100	1667
4	740	720	700	670	640	610	580	530	490	440	370	160	2333
5	800	800	800	770	740	710	660	610	570	510	440	230	3000
6	780	790	780	770	780	800	790	700	650	590	510	320	3667
7	700	710	720	730	750	750	750	750	730	670	600	400	4333
8	580	590	600	610	640	660	700	720	730	730	760	520	5000
9	490	490	490	480	490	510	600	650	660	690	580	450	5667
10	470	460	420	400	420	420	440	490	580	590	560	430	6333
11	340	340	340	340	340	330	350	390	450	280	380	370	7000
12	210	220	230	250	250	250	280	310	340	370	250	200	7667
13	160	150	140	120	130	150	200	230	280	290	230	110	8333
14	110	90	60	20	20	20	70	150	230	240	150	90	9000
15	50	20						40	140	180	90	50	9667
16									30	90	70	20	10333

8. Depreſſions below o in the S.E. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	165	165	160	155	150	140	130	120	110	100	95	40	$333\frac{1}{3}$
2	400	400	400	400	400	390	380	360	330	300	250	110	1000
3	600	610	610	610	610	600	600	580	550	500	440	200	1667
4	760	750	740	740	740	730	720	710	680	640	560	300	2333
5	800	800	800	800	800	800	800	800	790	740	660	400	3000
6	780	780	780	780	780	790	800	840	880	850	750	470	3667
7	700	690	680	670	670	680	620	720	820	900	770	520	4333
8	580	570	570	570	570	580	590	600	660	800	800	600	5000
9	490	490	490	490	490	490	490	500	520	700	880	600	5667
10	470	460	450	440	430	420	410	430	470	530	880	680	6333
11	340	330	320	320	320	320	330	350	420	500	780	780	7000
12	210	200	200	200	210	220	240	280	390	480	680	900	7667
13	120	120	130	130	140	150	180	230	300	450	600	990	8333
14	110	110	110	120	130	150	160	200	280	440	580	980	9000
15	70	70	70	70	90	120	140	170	240	420	570	990	9667
16	10	20	30	40	50	80	120	160	220	400	550	1000	10333
17					10	40	90	140	200	340	540	950	11000
18						5	40	110	170	300	500	850	11667
19								50	150	280	470	780	12333
20								20	150	250	400	700	13000

4. Altitudes above o in the S.E. quarter.

Rings.	1	2	3	4	5	6	7	8		Radii.
16	10									10333
17	80	60	50	50	10					11000
18	220	200	180	130	70	30				11667
19	340	300	260	240	[illegible]	120	20			12333
20	450	410	380	330	260	180	100	20		13000

5. Depreſſions below o in the N.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
6	70	40	15	5								15	3667
7	250	240	200	150	60	30						40	4333
8	460	450	430	390	280	200	80	10				80	5000
9	700	700	680	630	520	450	340	170	15			120	5667
10	840	830	800	780	650	600	520	380	180	40	70	220	6333
11	960	920	880	850	750	650	630	550	350	250	300	430	7000
12	1100	1000	950	900	820	780	780	780	580	530	500	560	7667
13	1130	1080	980	880	840	800	830	860	780	690	630	640	8333
14	1180	1100	1000	900	900	900	910	940	870	800	700	500	9000
15	1180	1100	1100	1080	1040	1050	1060	1070	1000	870	730	300	9667
16	1100	1100	1100	1100	1100	1140	1150	1150	1120	990	760	160	10333
17	1100	1100	1100	1130	1180	1200	1200	1200	1180	1080	700	80	11000
18	1100	1100	1150	1200	1200	1150	1100	1100	1200	1180	700	100	11667
19	1100	1120	1220	1230	1260	1200	1200	1200	1300	1240	620	60	12333
20	1120	1220	1320	1360	1390	1390	1390	1340	1440	1300	620	50	13000

6. Depreſſions below o in the N.E. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
6	70	60	30	10								10	3667
7	260	240	200	150	110	80	30	40	30	10		10	4333
8	450	440	400	350	280	180	100	180	180	190	190	40	5000
9	700	690	680	610	520	400	200	240	290	350	330	200	5667
10	850	870	890	860	770	620	440	300	380	500	450	370	6333
11	1020	1060	1070	1050	980	[illegible]	700	520	400	650	600	530	7000
12	1140	1160	1180	1160	1140	1080	950	840	620	720	850	700	7667
13	1200	1190	1200	1220	1240	1250	1160	1050	900	840	950	880	8333
14	1230	1130	1050	1050	1100	1220	1260	1220	1070	950	1020	990	9000
15	1100	960	900	850	900	1100	1230	1210	1170	1060	1090	1100	9667
16	970	860	880	[illegible]	780	900	1120	1180	1200	1180	1160	1150	10333
17	970	800	760	750	750	780	1000	1200	1300	1240	1200	1100	11000

7. Depreſſions below o in the S.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	165	165	160	155	150	140	130	120	110	90	50	10	333
2	400	390	380	370	350	330	300	270	240	210	160	60	1000
3	600	580	560	530	500	570	540	400	370	340	280	100	1667
4	740	720	700	670	640	610	580	530	490	440	370	160	2333
5	800	800	800	770	740	710	660	610	570	510	440	230	3000
6	780	790	780	770	780	800	790	700	650	590	510	320	3667
7	700	710	720	730	750	750	750	750	730	670	600	400	4333
8	580	590	600	610	640	660	700	720	730	730	760	520	5000
9	490	490	490	480	490	510	600	650	660	690	580	450	5667
10	470	460	420	400	420	420	440	490	580	590	560	430	6333
11	340	340	340	340	340	330	350	390	450	480	380	370	7000
12	210	220	230	250	250	250	280	310	340	370	250	200	7667
13	160	150	140	120	130	150	200	230	280	290	230	110	8333
14	110	90	60	20	20	20	70	150	230	240	150	90	9000
15	50	20						40	140	180	90	50	9667
16									30	90	70	20	10333

8. Depreſſions below o in the S.E. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	165	165	160	155	150	140	130	120	110	100	95	40	$333\frac{1}{3}$
2	400	400	400	400	400	390	380	360	330	300	250	110	1000
3	600	610	610	610	610	600	600	580	550	500	440	200	1667
4	760	750	740	740	740	730	720	710	680	640	560	300	2333
5	800	800	800	800	800	800	800	800	790	740	660	400	3000
6	780	780	780	780	780	790	800	840	880	850	750	470	3667
7	700	690	680	670	670	680	620	720	820	900	770	520	4333
8	580	570	570	570	570	580	590	600	660	800	800	600	5000
9	490	490	490	490	490	490	490	500	520	700	880	600	5667
10	470	460	450	440	430	420	410	430	470	530	880	680	6333
11	340	330	320	320	320	320	330	350	420	500	780	780	7000
12	210	200	200	200	210	220	240	280	390	480	680	900	7667
13	120	120	130	130	140	150	180	230	300	450	600	990	8333
14	110	110	110	120	130	150	160	200	280	440	580	980	9000
15	70	70	70	70	90	120	140	170	240	420	570	990	9667
16	10	20	30	40	50	80	120	160	220	400	550	1000	10333
17					10	40	90	140	200	340	540	950	11000
18						5	40	110	170	300	500	850	11667
19								50	150	280	470	780	12333
20								20	150	250	400	700	13000

9. Altitudes above P in the N.W. quarter.

Rings.		12	Radii.
4		10	2333
5		15	3000
6		15	3667
7		15	4333
8		15	5000

10. Altitudes above P in the N.E. quarter.

Rings.		12	Radii.
1		10	333⅓
2		10	1000
3		15	1667
4		60	2333
5		40	3000
6		5	3667

11. Altitudes above P in the S.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	110	110	105	105	100	100	95	95	90	85	80	35	333⅓
2	340	330	320	310	300	290	280	270	240	210	170	90	1000
3	660	660	660	660	660	650	620	590	570	510	370	170	1667
4	1020	1030	1040	1050	1060	1070	1030	990	910	800	660	270	2333
5	1030	1110	1280	1270	1320	1330	1310	1280	1270	1170	910	460	3000
6	670	770	810	900	930	950	1020	1070	1150	1270	1100	600	3667
7	280	340	420	480	540	570	620	670	720	880	1050	660	4333
8	20	50	90	140	210	290	350	420	490	570	700	570	5000
9							120	210	270	320	370	270	5667
10									90	150	180	20	6333
15									140	170			9667
16									140	470	30		10333
17									130	500	170	170	11000
18								20	170	600	400	400	11667
19	150	150	140	120	90	60	30	70	170	500	500	500	12333
20	160	170	210	220	200	170	100	130	170	400	600	600	13000

12. Altitudes above P in the S.E. quadrant.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	110	110	105	105	100	100	95	95	90	85	80	35	333⅓
2	340	330	320	310	300	290	280	270	240	210	170	90	1000
3	660	640	620	600	570	540	510	480	440	380	290	160	1667
4	1000	980	950	910	870	810	730	670	540	460	330	170	2333
5	1020	1020	1020	1030	1030	1020	970	770	570	470	390	130	3000
6	670	710	770	810	840	860	910	890	720	650	400	30	3667
7	290	320	360	390	470	590	700	750	780	600	280		4333
8				20	70	170	250	420	630	550	170		5000
9								110	360	420	170		5667
10									70	180	120		6333
18		20	40	50	40								11667
19	100	100	100	100	80	30							12333
20	130	130	130	120	110	80							13000

13. Depreſſions below P in the N.W. quarter.

Rings.	1	2	3	4	5	6	7	8	9	10	11	12	Radii.
1	100	95	90	85	80	75	70	60	50	40	30	15	333⅓
2	390	380	360	330	310	290	270	240	210	180	150	60	1000
3	520	510	500	490	480	470	450	430	410	370	270	80	1667
4	650	640	620	610	590	570	550	530	500	460	390	90	2333
5	830	820	760	720	690	660	630	590	560	500	380	130	3000
6	880	860	850	790	730	700	670	640	580	480	340	280	3667
7	910	900	860	830	790	720	630	620	540	550	440	185	4333
8	930	890	840	800	830	710	610	610	580	530	520	430	5000
9	830	830	830	830	830	830	760	700	670	620	600	330	5667
10	730	740	755	770	785	800	815	830	780	750	720	460	6333
11	730	750	780	800	830	860	860	880	880	860	820	530	7000
12	750	770	810	860	910	930	950	960	950	930	880	580	7667
13	770	840	910	950	990	1030	1050	1050	1030	980	950	650	8333

fore be found by barely dividing the given perpendicular by the baſe; and if we find this number in its proper column in a table of ſines and tangents, on the ſame line with it, in the column of ſines will be found the ſine of the angle required. This ſeems to be the eaſieſt way of reſolving all the triangles when computed ſeparately. But as the labour would be very great in performing ſo many hundreds of arithmetical diviſions, &c. either by logarithms, or by the natural numbers, inſtead of it, the following method, propoſed by the Hon. Mr. CAVENDISH, was adopted, as being a much more expeditious way of obtaining the ſum of the ſines required. This method conſiſts in finding, in a very eaſy manner, the difference between each tangent and its correſponding ſine, from the given baſe and perpendicular, and then, ſubtracting the ſum of all the differences from the ſum of the tangents, there remains the ſum of the ſines. Several advantages attend this method of proceeding: for, to find the tangents we need not divide every perpendicular ſeparately by its correſponding baſe, but add together all the perpendiculars that are on the ſame line, and divide their ſum by their common baſe, which is the radius of the middle of the ring, and is placed on the ſame line with them towards the right-hand; for thus we ſhall have little more than a twelfth part of the number of diviſions to perform: alſo a great part of the tangents are ſo ſmall that they do not at all differ from their correſponding ſines in the number of decimals that it is neceſſary to continue the computations to, in all which caſes the trouble of finding the differences is ſaved; and thoſe differences which it is neceſſary to compute, are very readily found by inſpection on a peculiar kind of ſliding rule, which was conſtructed for this purpoſe, and of which I ſhall here give a ſhort deſcription.

This rule (the figure of which is repreſented tab. x. fig. 2.) conſiſts of three columns; one marked AF or baſe, which is moveable by ſliding it up or down by the ſide of the other two which are fixed; of theſe two the one contains the perpendicular altitudes or depreſſions, and the other the differences between the ſines and tangents to the radius 1. To conſtruct the numbers on this rule; form a ſeries of logarithmic tangents in arithmetical progreſſion, of which the firſt term is 9·000, and the common difference ·025; take out from a table the correſponding natural tangents, and place them in the firſt and ſecond columns of baſe and perpendicular, and the difference between the natural ſine and natural tangent in the laſt column, marked Diff. To make uſe of this ſcale; look out any baſe and its correſponding perpendicular in their proper columns, that is, any radius and its correſponding altitude or depreſſion in the ſixteen foregoing tables, without regarding the number of places they contain, and bring them to correſpond; then, if they conſiſt of the ſame number of places, the lower index on the ſlider or firſt column, or that anſwering to 1000, points to the true difference between the ſine and tangent in the laſt column; but if the number of places in the baſe exceed that in the perpendicular by one, the upper index 100 muſt be uſed. And in this manner were computed all the differences which were neceſſary to be found, and placed in their proper ſquares formed by the meeting of the horizontal and vertical lines, or rings and ſectoral ſpaces, in the following ſet of ſixteen tables, which correſpond to the foregoing ſet of ſixteen, each to each, according to the number of them, and marked at the tops with the numbers 1, 2, 3 &c. to 12 for the ſectoral ſpaces, and with the number of the rings on the left-hand margin. Alſo, in the column immediately after the number of the ring are placed the radii which formed the laſt column in the preceding tables; then, in the third column, are placed the ſums of the altitudes and depreſſions found in each line of the former tables; and, in the next column, the quotients found by dividing the numbers in the third by thoſe in the ſecond column; theſe quotients are the ſums of the tangents belonging to each line or ring, which being all added together, their total is placed at the bottom of the column: after this follow the twelve columns of differences before mentioned, which are ſucceeded by one more column containing the ſums of each line of theſe differences, which ſums being added together, their total is placed at the bottom of them; and this total is the ſum of all the differences between the ſines and the tangents, and it is therefore ſubtracted from the total of the tangents in the fourth column, when there remains the ſum of the ſines as required.

1. For the ſum of the ſines of alt. above o in the N.W. quarter.

Rings.	Radii.	Sum of Perpen.	3÷2= Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333⅓	2175	6·525	103	103	103	103	97	97	86	75	55	36	20	5	·883
2	1000	6265	6·265	88	90	88	85	84	82	78	75	56	40	20	4	790
3	1667	10045	6·027	79	86	87	87	88	97	93	64	48	21	7	1	758
4	2333	9300	3·986	12	12	13	17	23	27	39	43	45	16	4		251
5	3000	7275	2·425		1	1	2	3	6	11	16	27	13	3		83
6	3667	4695	1·280						1	1	3	7	9	2		23
7	4333	2735	0·631								1	1	3	1		6
8	5000	1625	0·325										1	1		2
9	5667	670	0·118													
10	6333	60	0·009													

27·591 = ſum of tangents. Sum of diff. 2·796
2·796 = ſum of the diff.
24·795 = ſum of the ſines of alt. above o in the N.W. quarter.

2. For the ſum of the ſines above o in the N.E. quarter.

Rings.	Radii.	Sum of Perp.	3÷2= Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333⅓	1965	5·895	97	92	92	87	80	71	56	43	33	24	15	5	·695
2	1000	5360	5·360	68	67	65	62	59	56	53	43	35	23	9	1	541
3	1667	8275	4·965	66	53	51	49	47	41	36	31	25	20	13	1	433
4	2333	7555	3·238	10	10	10	10	10	11	12	14	15	13	8	1	124
5	3000	4410	1·470					1	1	1	2	2	2	4		13
6	3667	1937	·528										1	1		2
7	4333	580	·134													
8	5000	50	·010													
			21·600 = ſum of the tangents.													1 808

1·808 = ſum of the differences.

19·792 = ſum of the ſines.

3. For the ſum of the ſines above o in the S.W. quarter.

1	333⅓	10	0·030	
2	1000	15	15	
3	1667	5	3	
12	7667	40	5	
13	8333	60	7	
14	9000	100	11	
15	9667	200	21	
16	10333	1960	190	
17	11000	3720	338	
18	11667	5770	495	
19	12333	7260	573	
20	13000	8920	686	

2·374 = ſum of the tangents, or ſum of the ſines, as the diff. between them are nothing in this quadrant.

4. For the ſum of the ſines above o in the S.E. quarter.

16	10333	10	0·001	
17	11000	230	21	
18	11667	830	71	
19	12333	1450	118	
20	13000	2130	164	

0·375 = ſum of the tangents, or of the ſines, the diff. being nothing.

5. For the ſines below o in the N.W. quarter.

Rings.	Radii.	Sum of Perp.	3÷2= Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
6	3667	145	0·040													
7	4333	970	224													
8	5000	2370	474													
9	5667	4325	763	1	1	1	1	1								5
10	6333	5910	933	1	1	1	1	1	1							6
11	7000	7520	1·074	1	1	1	1	1	1	1						7
12	7667	9280	1·210	2	1	1	1	1	1	1	1					9
13	8333	10140	1·219	2	1	1	1	1	1	1	1	1				10
14	9000	10700	1·078	1	1	1	1	1	1	1	1	1				9
15	9667	11580	1·198	1	1	1	1	1	1	1	1	1	1			10
16	10333	11970	1·159	1	1	1	1	1	1	1	1	1				9
17	11000	12250	1·105	1	1	1	1	1	1	1	1	1	1			10
18	11667	12280	1·052	1	1	1	1	1	1	1	1	1	1			10
19	12333	12750	1·034			1	1	1	1	1	1	1	1			8
20	13000	13940	1·072			1	1	1	1	1	1	1	1			8
			13·635 = ſum of the tangents.													·101

·101 = ſum of the differences.

13·534 = ſum of the ſines.

6. For the ſum of the ſines below o in the N.E. quarter.

				1	2	3	4	5	6	7	8	9	10	11	12	
6	3667	180	0·049													
7	4333	1160	268													
8	5000	2980	596													
9	5667	5210	919	1	1	1	1	1								5
10	6333	7300	1·153	1	1	1	1	1	1							6
11	7000	9440	1·349	1	2	2	2	1	1	1						10
12	7667	11460	1·495	2	2	2	2	2	1	1	1		1	1		15
13	8333	13080	1·570	1	1	1	1	2	2	1	1	1	1	1	1	14
14	9000	13290	1·477	1	1	1	1	1	1	1	1	1	1	1	1	12
15	9667	12670	1·311	1	1	1		1	1	1	1	1	1	1	1	11
16	10333	12160	1·177		1		1		1		1		1		1	6
17	11000	11850	1·077			1			1		1	1		1	1	6
			12·441 = ſum of the tangents.													·085

·085 = ſum of the differences.

12·356 = ſum of the ſines.

7. For the ſum of the ſines below 0 in the S.W. quarter.

Rings.	Radii.	Sum of Dep.	3÷2= Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	3 3⅓	1445	4·335	51	51	47	43	40	35	27	21	17	10	2		·344
2	1000	3460	3·460	28	27	26	25	19	17	13	9	6	5	2		177
3	1667	5370	3·222	21	19	17	15	13	18	16	6	5	4	2		136
4	2333	6650	2·850	15	14	13	12	10	8	8	5	5	3	2		95
5	3000	7640	2·547	9	9	9	8	6	6	5	4	3	3	2		61
6	3667	8260	2·253	5	5	5	5	5	5	5	3	3	2	1		44
7	4333	8260	1·906	2	2	2	2	3	3	3	3	2	2	1		25
8	5000	7840	1·568	1	1	1	1	1	1	1	1	2	2	2	1	15
9	5667	6580	1·161	1	1	1	1	1	1	1	1	1	1	1		11
10	6333	5680	897	1							1	1	1	1		5
11	7000	4450	636				1				1				1	3
12	7667	3160	412						1						1	2
13	8333	2190	263										1			1
14	9000	1250	139													
15	9667	570	59													
16	10333	210	20													·922

25·728 = ſum of the tangents.
0·922 = ſum of the differences.
24·806 = ſum of the ſines.

8. For the ſum of the ſines below o in the S.E. quarter.

Rings.	Radii.	Sum of Dep.	3÷2= Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333⅓	1510	4·530	51	51	47	43	40	35	27	21	17	13	11	1	·357
2	1000	4120	4·120	28	28	28	28	28	27	25	21	17	13	8	1	252
3	1667	6510	3·906	21	21	21	21	21	21	2	19	16	13	9	1	205
4	2333	8070	3·459	16	16	15	15	15	15	14	14	12	10	6	1	15
5	3000	8990	2·997	9	9	9	9	9	9	9	9	9	8	5	1	95
6	3667	9280	2·531	5	5	5	5	5	5	5	5	6	6	4	1	57
7	4333	8440	1·948	2	2	2	2	2	2	2	2	3	4	3	1	27
8	5000	7490	1·498	1	1	1	1	1	1	1	1	1	2	2	1	14
9	5667	6630	1·170			1		1				1	1	1	1	6
10	6333	6070	958			1			1			1		1	1	5
11	7000	5110	730				1				1		1	1	1	5
12	7667	4210	549						1				1		1	3
13	8333	3540	425								1			1	1	3
14	9000	3370	374								1			1	1	3
15	9667	3020	313										1		1	2
16	10333	2680	259											1	1	2
17	11000	2310	210												1	1
18	11667	1975	169												1	1
19	12333	1730	140												1	1
20	13000	1520	117												1	1
																1·190

30·403 = ſum of the tangents.
1·190 = ſum of the differences.
29·213 = ſum of the ſines.

9. For the ſum of the ſines above P in the N.W. quarter.

4	2333	10	·004	
5	3000	15	5	
6	3667	15	4	
7	4333	15	3	
8	5000	15	3	

0·019 = ſum of the tangents, or ſum of the ſines, as they have no difference in this quadrant.

10. For the ſum of the ſines above P in the N.E. quarter.

1	333⅓	10	·030	
2	1000	10	10	
3	1667	15	9	
4	2333	60	26	
5	3000	40	13	
6	3667	5	2	

0·090 = ſum of the tangents, or ſum of the ſines, they being equal in this quadrant.

11. For the ſum of the ſines above P in the S.W. quarter.

Rings.	Radii.	Sum of Alt.	3÷2 = Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333⅓	1110	3·330	17	17	15	15	13	13	12	12	10	8	6	1	·139
2	1000	3150	3·150	18	17	16	14	13	12	11	10	6	5	2	1	125
3	1667	6780	4·068	28	28	28	28	28	27	23	20	18	14	5	1	248
4	2333	10930	4·684	37	38	39	40	41	42	38	34	26	18	7	1	361
5	3000	13740	4·580	18	23	34	33	38	39	37	34	33	26	14	2	331
6	3667	11240	3·065	3	5	5	6	6	7	14	16	19	21	14	2	118
7	4333	7230	1·668			1	1	1	1	1	2	2	4	5	2	20
8	5000	3900	780							1		1	1	1	1	5
9	5667	1560	275											1		1
10	6333	440	70													
15	9667	310	32													
16	10333	640	62													
17	11000	970	88										1			1
18	11667	1590	137										1			1
19	12333	2480	201												1	1
20	13000	3130	241										1		1	2
																1·353

26·431 = ſum of the tangents.
1·353 = ſum of the differences.
25·078 = ſum of the ſines.

12. For the ſum of the ſines above P in the S.E. quarter.

				1	2	3	4	5	6	7	8	9	10	11	12	
1	333⅓	1110	3·330	17	17	15	15	13	13	12	12	10	8	6	1	·139
2	1000	3150	3·150	18	17	16	14	13	12	11	10	6	5	2	1	125
3	1667	5890	3·534	28	26	23	21	18	16	14	12	9	5	2	1	175
4	2333	8420	3·609	34	33	30	26	24	18	15	12	6	4	1		203
5	3000	9440	3·147	18	18	18	18	18	18	16	9	3	2	1		139
6	3667	8260	2·253	3	4	5	5	5	6	8	7	4	3	1		51
7	4333	5530	1·276			1		1	1	2	3	3	1			12
8	5000	2280	456								1	1	1			3
9	5667	1060	187										1			1
10	6333	370	58													
18	11667	150	13													
19	12333	510	42													
20	13000	700	54													
																0·848

21·109 = ſum of the tangents.
0·848 = ſum of the differences.
20·261 = ſum of the ſines.

13. For the ſum of the ſines below P in the N.W. quarter.

Rings.	Radii.	Sum of Dep.	3÷2= Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
1	333⅓	790	2·370	13	12	9	8	7	5	5	3	2	1	1		066
2	000	3170	3·170	26	25	21	17	14	12	9	6	5	3	2		140
3	1667	4980	2·988	15	14	13	12	12	11	9	8	7	5	2		108
4	2333	6200	2·657	10	10	9	9	8	7	6	5	5	4	2		75
5	3000	7270	2·423	10	10	8	7	6	5	5	4	3	2	1		61
6	3667	7800	2·127	7	6	5	5	4	3	3	3	2	1	1		40
7	4333	7975	1·840	5	4	4	3	3	2	2	1	1	1	1		27
8	5000	8280	1·656	3	3	2	2	2	1	1	1	1	1			17
9	5667	8660	1·528	2	1	2	1	2	1	1	1	1	1	1		14
10	6333	8935	1·411	1	1	1	1	1	1	1	1	1	1	1		11
11	7000	9580	1·369	1		1		1	1	1	1	1	1	1		9
12	7667	10280	1·341	1		1		1	1	1	1	1	1	1		9
13	8333	11200	1·344		1	1	1	1	1	1	1	1	1	1		10
			26·224 = ſum of the tangents. 0·587 = ſum of the differences. 25·637 = ſum of the ſines.													0·587

14. For the ſum of the ſines below P in the N.E. quarter.

				1	2	3	4	5	6	7	8	9	10	11	12	
1	333⅓	625	1·875	7	5	5	4	3	2	2	1	1	1	1		·032
2	1000	2755	2·755	17	16	15	13	11	9	6	5	3	2	1		98
3	1667	4855	2·913	15	14	13	12	11	10	9	8	6	4	2		104
4	2333	7000	3·000	11	12	13	13	13	13	11	9	6	4	2		107
5	3000	8500	2·833	11	11	11	11	11	10	10	9	7	5	2		98
6	3667	9580	2·613	6	7	7	8	8	8	8	6	5	4	2		69
7	4333	10350	2·389	5	5	5	4	4	4	5	5	5	4	2	1	49
8	5000	11840	2·368	3	2	2	2	2	1	2	3	3	3	3	1	27
9	5667	8400	1·482	1	1	1	1	1	1		1	2	2	2	1	14
10	6333	7610	1·202	1	1		1		1		1	1	1	1	1	9
11	7000	7930	1·133	1		1		1		1			1	1	1	7
12	7667	8810	1·149	1		1		1		1	1		1	1	1	8
13	8333	8980	1·078		1		1		1		1		1	1	1	7
			26·790 = ſum of the tangents. ·629 = ſum of the differences. 26·161 = ſum of the ſines.													·629

15. For the ſum of the ſines below P in the S.W. quarter.

Rings.	Radii.	Sum of Dep.	3÷2= Sum of Tang.	1	2	3	4	5	6	7	8	9	10	11	12	Sum of Diff.
9	5667	620	·051													
10	6333	1730	273			1										·001
11	7000	3030	433	1						1						2
12	7667	4470	583		1							1				2
13	8333	4660	559				1				1					2
14	9000	3730	414				1							1		2
15	9667	2390	247							1						1
16	10333	1460	141								1					1
17	11000	800	73													
18	11667	130	11													
			2·785 = ſum of the tangents.													·011

·011 = ſum of the differences.

2·774 = ſum of the ſines.

16. For the ſum of the ſines below P in the S.E. quarter.

				1	2	3	4	5	6	7	8	9	10	11	12	
7	4333	80	·018													
8	5000	290	58													
9	5667	1810	319				1									·001
10	6333	3280	518			1				1						2
11	7000	4640	663		1				1						1	3
12	7667	5590	729			1			1						1	3
13	8333	5830	700				1			1					1	3
14	9000	5700	633				1				1			1		3
15	9667	5240	542							1			1		1	3
16	10333	4560	441								1			1		2
17	11000	3920	356									1			1	2
18	11667	3180	273											1		1
19	12333	2630	213											1		1
20	13000	2230	172											1		1
			5·635 = ſum of the tangents.													·025

·025 = ſum of the differences.

5·610 = ſum of the ſines.

Having now obtained the ſums of the ſines for the ſeveral quadrants, the next buſineſs is to collect them together, and deduct the negatives from the affirmatives. And this may be done either for each obſervatory ſeparately, or for both together. I ſhall do them ſeparately, in order thereby to diſcover alſo the ratio of their effects.

And, firſt, for the Southern obſervatory o.

Affirmatives.		Negatives.	
1..24·795 N.W.	Alt.	3 . . 2·374 S.W.	Alt.
2..19·792 N.E.		4 . . 0·375 S.E.	
7..24·806 S.W.	Dep.	5..13·534 N.W.	Dep.
8..29·213 S.E.		6..12·356 N.E.	
98·606 = ſum of affirm.		28·639 ſum.	

28·639 = ſum of negat.

69·967 = effective ſum of the ſines for o.

Secondly, for the Northern obſervatory P.

Affirmatives.		Negatives.	
11..25.078 S.W.	Alt.	9..0·019 N.W.	Alt.
12..20·261 S.E.		10..0·090 N.E.	
13..25·637 N.W.	Dep.	15..2·774 S.W.	Dep.
14..26·161 N.E.		16..5·610 S.E.	
97·137 = ſum of affirm.		8·493	

8·493 = ſum of negat.

88·644 = effective ſum of the ſines for P.

69·967 = the ſame for o.

158·611 = the ſum of the ſines for both obſerv.

From theſe numbers it appears, that the effect of the attraction at the Northern obſervatory is to that at the Southern one, nearly as 70 is to 89, or as 7 to 9 nearly. This difference is to be attributed chiefly to the effect of the hills on the South of the Southern obſervatory, which were conſiderably greater and nearer to it than thoſe on the back of the Northern obſervatory. For although the Southern obſervatory was placed 273 feet above the level of the Northern one, which removed it conſiderably more above the center of gravity of the hill than the latter was, it was at the ſame time placed conſiderably nearer than the other to the middle in a horizontal direction; ſo that probably the one difference nearly balances the other; and accordingly we find that the ſum of the affirmative altitudes for O is 44·587, and of thoſe for P 45·339, which differ by only a 45th part nearly.

It only remains now to multiply the ſum of the ſines by the common breadth of the rings, and by the common difference of the ſines of the angles made by the meridian and the ſeveral radii. It has already been obſerved, that the former is $666\frac{2}{3}$, and the latter $\frac{1}{12}$; therefore $\frac{1}{12} \times 666\frac{2}{3} = \frac{2000}{36} = \frac{500}{9}$ is their product: conſequently, $158{\cdot}611 \times \frac{500}{9} = 8811\frac{2}{3}$ nearly, is the ſum of the two oppoſite attractions made by the hill, &c. at the two obſervatories.

In order now to compare this attraction with that of the whole earth, this body may be conſidered as a ſphere, and the obſervatories as placed at its ſurface; ſince the very ſmall differences of theſe ſuppoſitions from the truth, are of no conſequence at all in this compariſon. Now the attraction of a ſphere, on a body at its ſurface, is known to be $= \frac{2}{3}cd$, where d is = the diameter of the ſphere, and $c =$ 3·1416 = the circumference of the circle of which the diameter is 1. But cd is = the circumference of the circle to the diameter d; and therefore the attraction of a ſphere will be expreſſed by barely $\frac{2}{3}$ of its circumference; which is a theorem well adapted to the computation in hand. The length of a degree in the mean latitude of 45°, is 57028 French toiſes (ſee p. 327. Phil. Tranſ. 1768): and the ſame reſult nearly is obtained by taking a mean among all the meaſures of degrees there put down, that mean being 57038 toiſes. I ſhall therefore uſe the round number 57030 as probably nearer the truth. This number being multiplied by 6, the product 342180 ſhews the number of French feet in one degree; but, by p. 326. of the ſame volume, the lengths of the Paris and London feet are as 76·734 to 72, that is, as 4·263 to 4; therefore, as 4 : 4·263 :: 342180 : 364678 = the Engliſh feet in one degree; and this being multiplied by 360 the whole number of degrees, there reſults

131284080

131284080 feet for the whole circumference, which are equal to $24864\frac{1}{2}$ miles, making $69\frac{1}{15}$ to a degreee in the mean latitude. Laſtly, $\frac{2}{3}$ of 131284080 give 87522720 for the meaſure of the attraction of the whole earth.

Conſequently, the whole attraction of the earth is to the ſum of the two contrary attractions of the hill, as the number 87522720 to $8811\frac{2}{3}$, that is, as 9933 to 1 very nearly, on ſuppoſition that the denſity of the matter in the hill is equal to the mean denſity of that in the whole earth.

But the Aſtronomer Royal found, by his obſervations, that the ſum of the deviations of the plumb line, produced by the two contrary attractions, was 11·6 ſeconds. From hence it is to be inferred, that the attraction of the earth is actually to the ſum of the attractions of the hill, nearly as radius to the tangent of 11·6 ſeconds, that is, as 1 to ·000056239, or as 17781 to 1; or as 17804 to 1 nearly, after allowing for the centrifugal force ariſing from the rotation of the earth about its axis.

Having now obtained the two reſults, namely, that which ariſes from the actual obſervations, and that belonging to the computation on the ſuppoſition of an equal denſity in the two bodies, the two proportions compared muſt give the ratio of their denſities, which is that of 17804 to 9933, or 1434 to 800 nearly, or almoſt as 9 to 5. And ſo much does the mean denſity of the earth exceed that of the hill.

Thus then we have at length obtained the object which we have been in queſt of through the very laborious calculations that have been deſcribed in this paper, and in the ſurvey and meaſurements from which theſe computations were made; namely, the ratio of the mean denſity of all the matter in the earth, in compariſon with the denſity of the matter of which the hill is compoſed. And that ratio we have found to be equal to the ratio of 9 to 5. And, for the reaſons before mentioned, I think we may reſt ſatisfied, that this proportion is obtained to a conſiderable degree of proximity, probably to within the fiftieth part, if not the hundredth part of its true magnitude. Another queſtion, however, ſtill ariſes, namely, what is the denſity of the matter in the hill? Is its mean denſity equal to that of water, of ſand, of clay, of chalk, of ſtone, or of ſome of the metals? For, according to the matter, or different ſorts of matter, of which it is formed, and according as it is conſtituted with or without large vacuities, its mean denſity may be greater or leſs, and that in a degree which is not certainly known. A conſiderable degree of accuracy in this point could only be obtained by a cloſe examination of the internal ſtructure of the hill. And the eaſieſt method of doing this would be to procure holes to be bored, in ſeveral parts of it, from the ſurface to a ſufficient depth, after the manner that is practiced in boring holes to the coal mines from the ſurface of the ground; for by ſuch operation it is known what kind of ſtrata the borer is paſſed through, together with their dimenſions and denſities. The proper mean among all theſe would be the mean denſity of the hill, as compared to water or to any other ſimple matter; and thence we ſhould obtain the comparative denſity of the whole earth with reſpect to water: but in the preſent inſtance, we muſt be ſatisfied with the eſtimate ariſing from the report of the external view of the hill; which is, that to all appearance it conſiſts of an intire maſs of ſolid rock. It is probable, therefore, that we ſhall not greatly err, if we aſſume the denſity of the hill equal to that of common ſtone; which is not much different from the mean denſity of the whole matter near the ſurface of the earth, to ſuch depths as have actually been explored either by digging or boring. Now the denſity of common ſtone is to that of rain water as $2\frac{1}{2}$ to 1; which being compounded with the proportion of 9 to 5 above found, there reſults the ratio of $4\frac{1}{2}$ to 1 for the ratio of the denſities of the earth and rain water;

that is to ſay, the mean denſity of the whole earth is about $4\frac{1}{2}$ times the denſity of water.

To what uſeful purpoſes the knowledge of the mean denſity of the earth, as above found, may be applied, it is not my buſineſs here to ſhew. I ſhall therefore put an end to this paper with a reflection or two on the premiſes before delivered. Sir ISAAC NEWTON thought it probable, that the mean denſity of the earth might be five or ſix times as great as the denſity of water; and we have now found, by experiment, that it is very little leſs than what he had thought it to be: ſo much juſtneſs was even in the ſurmiſes of this wonderful man! Since then the mean denſity of the whole earth is about double that of the general matter near the ſurface, and within our reach, it follows, that there muſt be ſomewhere within the earth, towards the more central parts, great quantities of metals, or ſuch like denſe matter, to counterbalance the lighter materials, and produce ſuch a conſiderable mean denſity. If we ſuppoſe, for inſtance, the denſity of metal to be 10, which is about a mean among the various kinds of it, the denſity of water being 1, it would require ſixteen parts out of twenty-ſeven, or a little more than one-half of the matter in the whole earth, to be metal of this denſity, in order to compoſe a maſs of ſuch mean denſity as we have found the earth to poſſeſs by our experiment: or $\frac{4}{15}$, or between $\frac{1}{3}$ and $\frac{1}{4}$ of the whole magnitude will be metal; and conſequently $\frac{20}{31}$, or nearly $\frac{2}{3}$ of the diameter of the earth, is the central or metalline part.

Knowing then the mean denſity of the earth in compariſon with water, and the denſities of all the planets relatively to the earth, we can now aſſign the proportions of the denſities of all of them as compared to water, after the manner of a common table of ſpecific gravities. And the numbers expreſſing their relative denſities, in reſpect of water, will be as below, ſuppoſing the denſities of the planets, as compared to each other, to be as laid down in Mr. DE LA LANDE'S aſtronomy.

Water . . .	1
The Sun . .	$1\frac{2}{15}$
Mercury . .	$9\frac{1}{6}$
Venus . . .	$5\frac{11}{15}$
The earth . .	$4\frac{1}{2}$
Mars	$3\frac{2}{7}$
The Moon . .	$3\frac{1}{14}$
Jupiter . . .	$1\frac{1}{24}$
Saturn . . .	$\frac{13}{32}$

Thus then we have brought to a concluſion the computation of this important experiment, and, it is hoped, with no inconſiderable degree of accuracy. But it is the firſt experiment of the kind which has been ſo minutely and circumſtantially treated; and firſt attempts are ſeldom ſo perfect and juſt as ſucceeding endeavours afterwards render them. And, beſides, a frequent repetition of the ſame experiment, and a coincidence of reſults, afford that firm dependance on the concluſions and ſatiſfaction to the mind, which can ſcarcely ever be had from a ſingle trial, however carefully it may be executed. For thoſe reaſons it is to be wiſhed, that the world may not reſt ſatisfied barely with what has been done in this inſtance, but that they will repeat the experiment in other ſituations, and in other countries, with all the care and preciſion that it may be poſſible to give to it, till an uniformity of concluſions ſhall be found, ſufficient to eſtabliſh the point in queſtion beyond any reaſonable poſſibility of doubt. What has been already done in the preſent caſe will render any future repetition more eaſy and perfect. But improvements may be made, perhaps both in the mode of computation and in the ſurvey; in the latter, eſpecially, there certainly may. Some improvements of this kind I have hinted at in ſome parts of this paper, which with others I ſhall here collect together, that they may readily be ſeen in one point of view. They are principally theſe. Procure one baſe, or more if convenient, very accurately meaſured, in ſuch ſituation, that as many more points as poſſible in the ſurvey may be ſeen from it. Aſſume as many principal or eminent points and objects as may be proper and convenient; and from each one of them meaſure the angles formed by all the reſt that can be ſeen, both horizontal and vertical angles, and repeat theſe obſervations, if convenient, with the inſtrument varied or reverſed, taking the means among the ſeveral quantities of each angle. Take then as many ſections of the ground, and as far extended in all directions, as the time and circumſtances will poſſibly admit. Of the ſections, thoſe that are horizontal or level are the beſt, as they require no calculation; procure therefore as many as poſſible of them. In vertical ſections obſerve the vertical angles, not in the plane of the ſection, but at ſome other point of which the bearing is alſo taken from the beginning of the ſection line, and where the horizontal angles of the poles are taken, for the reaſons before mentioned in p. 35. And it will be a ſtill farther convenience if the ſection be made in ſuch direction as to form a right angle with the line drawn to the point or ſtation from whence the vertical angles of the poles are obſerved, as may be ſeen from what is ſaid in p. 33. It might, perhaps, be proper to make ſome experiments on a valley inſtead of a hill, taking two obſervatories at the two oppoſite ſides of it, both for the greater variety in this intereſting problem, and becauſe alſo the ſurvey would be more eaſily made, on account of the ground being more in view at each ſtation than in the caſe of a hill, which generally hides more than half the compaſs from the obſerver. In computing the relative altitudes of all the principal ſtations, let the operations be performed mutually both backwards and forwards, that is, from both of every two objects, having for that purpoſe obſerved at each of them the vertical angle of the other, namely, both the angle of elevation and the angle of depreſſion, and take the mean between the two computed differences of altitude; for this excludes the neceſſity of making the proper allowances for refraction, and for the curvature of the earth; ſince the effect of each of theſe is balanced and corrected by that of the counter obſervation. But as to thoſe points in the ſections which are far diſtant from the obſerver, and where great accuracy is required, it may be proper to make the allowance for refraction and curvature, as there is generally no back obſervation by which their effects may be balanced. Theſe are the chief hints which at preſent occur to me, beſides the general information to be derived by the computer from the peruſal of the modes of computation that have been deſcribed in this paper. As to the ſurveyor, he will ſtrike out other convenient ways of meaſurement adapted to the circumſtances with which the nature of the ſurvey may happen to be attended.

A map of the country about Schehallien is hereunto annexed, to convey a general idea of the nature of the ground, and for the better illuſtration of the deſcription given in the former parts of this paper. This map is tab. XI.

Woolwich,
April 27, 1778.

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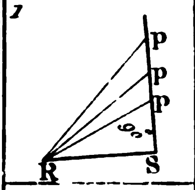

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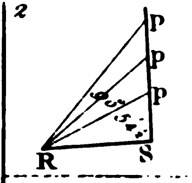

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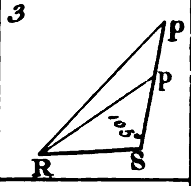

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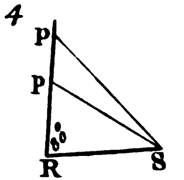

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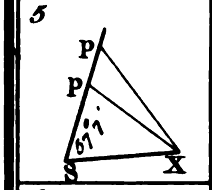

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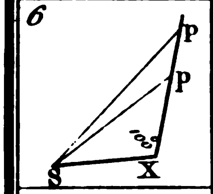

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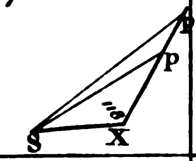

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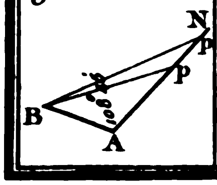

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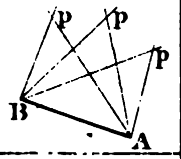

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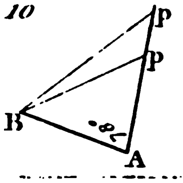

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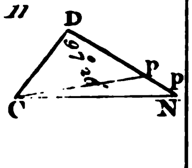

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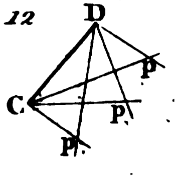

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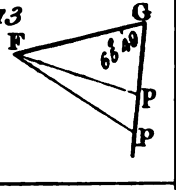

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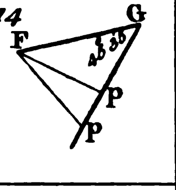

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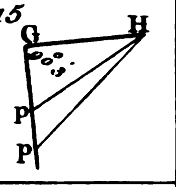

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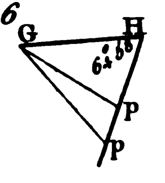

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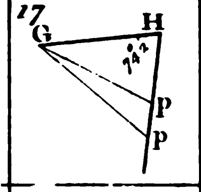

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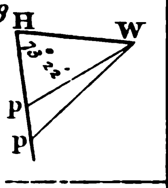

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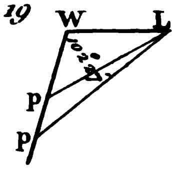

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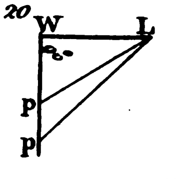

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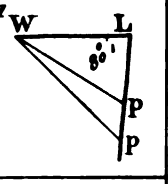

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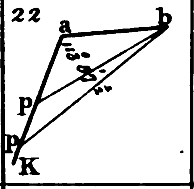

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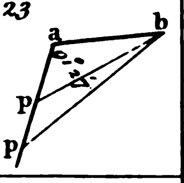

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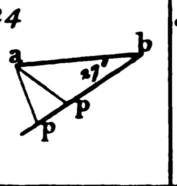

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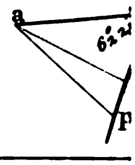

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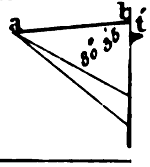

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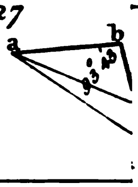

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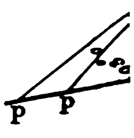

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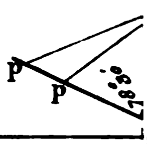

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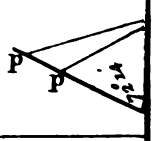

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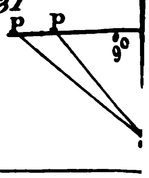

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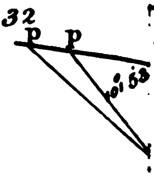

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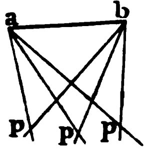

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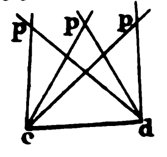

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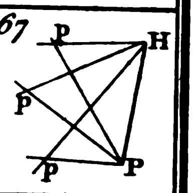

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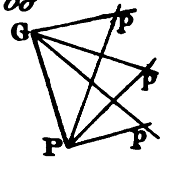

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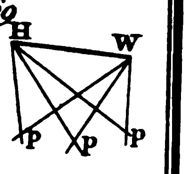

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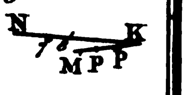

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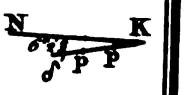

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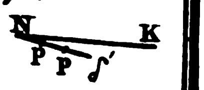

Basire sc.

Fig. 1.

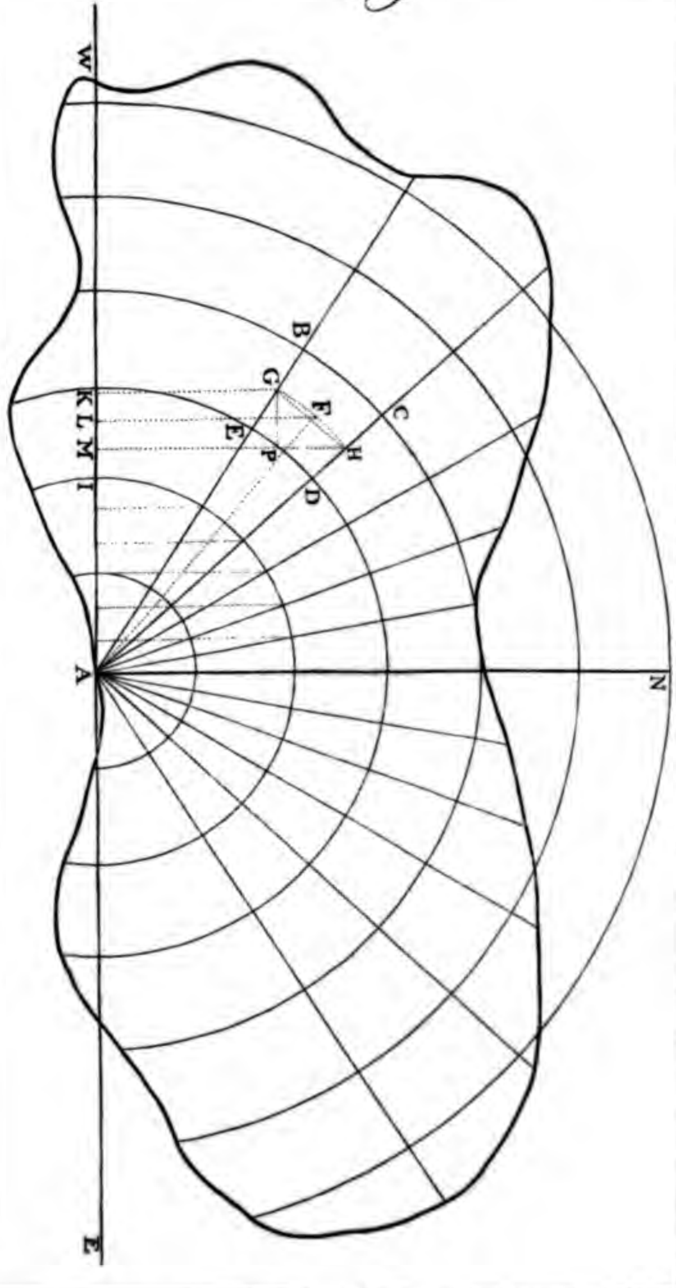

Fig. 2.

A.F or Base	Perpendicular	Diff.
100	100	·001
106	106	·001
112	112	·001
119	119	·001
126	126	·001
133	133	·001
141	141	·001
150	150	·002
159	159	·002
168	168	·002
178	178	·003
188	188	·003
199	199	·004
211	211	·005
224	224	·005
237	237	·006
251	251	·008
266	266	·009
282	282	·011
299	299	·013
316	316	·015
335	335	·017
355	355	·020
376	376	·024
398	398	·028
422	422	·033
447	447	·039
473	473	·045
501	501	·053
531	531	·062
562	562	·072
596	596	·084
631	631	·097
668	668	·113
708	708	·130
750	750	·150
794	794	·172
842	842	·198
891	891	·226
944	944	·258
1000	1000	·293

Basire sc.

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Fig. 1.

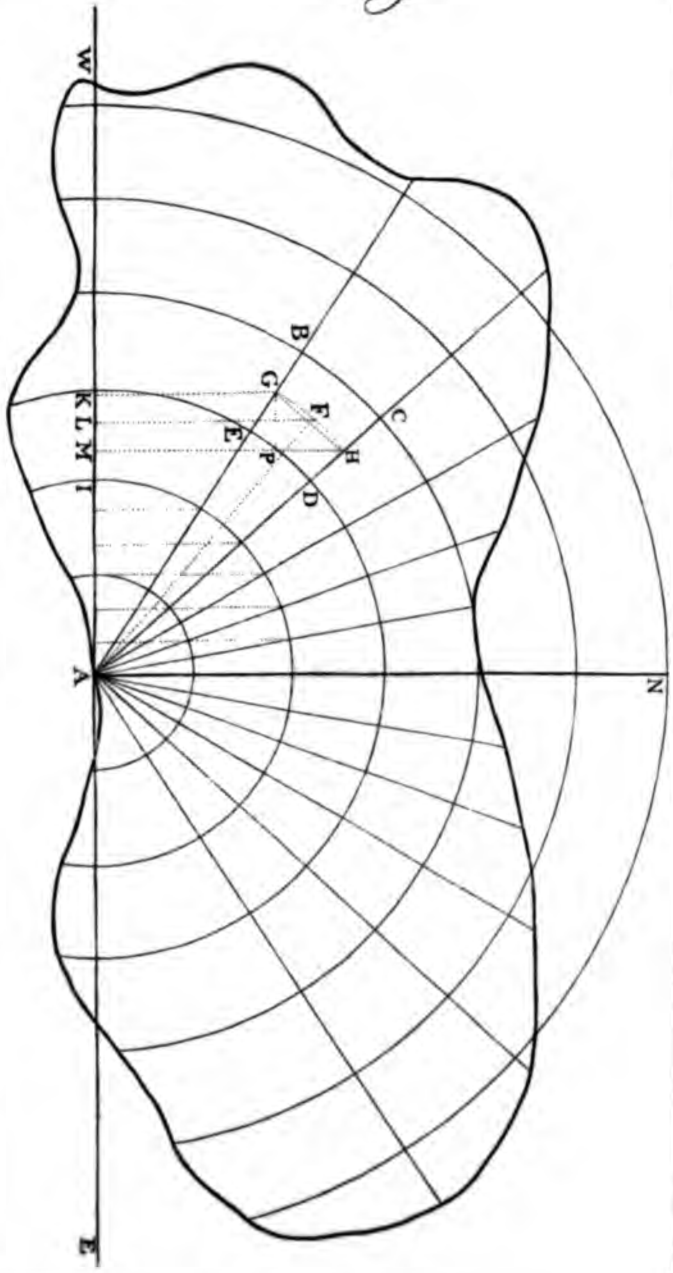

Fig. 2.

A.F *or Base*	*Perpendicular*	*Diff.*
100	100	·001
106	106	·001
112	112	·001
119	119	·001
126	126	·001
133	133	·001
141	141	·001
150	150	·002
159	159	·002
168	168	·002
178	178	·003
188	188	·003
199	199	·004
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237	237	·006
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335	335	·017
355	355	·020
376	376	·024
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422	422	·033
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473	473	·045
501	501	·053
531	531	·062
562	562	·072
596	596	·084
631	631	·097
668	668	·113
708	708	·130
750	750	·150
794	794	·172
842	842	·198
891	891	·226
944	944	·258
1000	1000	·293